

# 國立臺北大學 103 學年度碩士班一般入學考試試題

系(所)組別：統計學系  
科 目：基礎數學

第 1 頁 共 1 頁  
可 不可使用計算機

1. Let  $f(x) = \left(1 + \frac{1}{x}\right)^x$ ,  $x > 0$ .

(a) Find  $f'(x)$ . (7%)

(b) Evaluate  $\lim_{x \rightarrow 0^+} f(x)$  and  $\lim_{x \rightarrow \infty} f(x)$ . (8%)

2. Approximate the value of  $\int_1^2 \frac{1}{x} dx$  by Simpson's Rule for  $n=4$ . (10%)

3. Find the area of the region bounded by  $y = x$ ,  $y = 4x$ ,  $xy = 1$ , and  $xy = 3$ . (10%)

4. Evaluate the following integrals.

(a)  $\int_0^{\infty} x^3 e^{-x^2} dx$  (7%)

(b)  $\int_0^1 \int_y^1 e^{-x^2} dx dy$  (8%)

5. Please orthogonally diagonalize the matrix  $A = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$  (That is, find an orthogonal matrix P and a diagonal

matrix D such that  $D = P^T A P$ ). (12%)

6. Let  $\lambda_1, \lambda_2, \dots, \lambda_n$  be distinct eigenvalues of a matrix A with associated eigenvectors  $\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n$ . Please show that  $\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n$  are linear independent. (12%)

7. Let A, B, C are  $n \times n$  matrices and  $BA = CA$ . Show that if  $\det(A) \neq 0$ , then  $B = C$ . (7%)

8. Please solve the linear system

$$\begin{cases} x + 2y + 3z + 4w = 5 \\ x + 3y + 5z + 7w = 11 \\ x - z - 2w = -7 \end{cases} \quad (7\%)$$

9. Let  $V$  be the vector space with basis  $S = \{\cos\theta, \sin\theta\}$ , and let  $T = \{\sin\theta - \cos\theta, \sin\theta + \cos\theta\}$  be another basis for  $V$ . Find the matrix of the linear operator  $L: V \rightarrow V$  defined by  $L(f) = f'$  with respect to (a)  $S$  (b)  $T$  (c)  $S$  and  $T$ . (12%)