國立臺灣科技大學103學年度碩士班招生試題

系所組別： 工業管理系碩士班甲組

科　　目： 作業研究

（總分為100分）

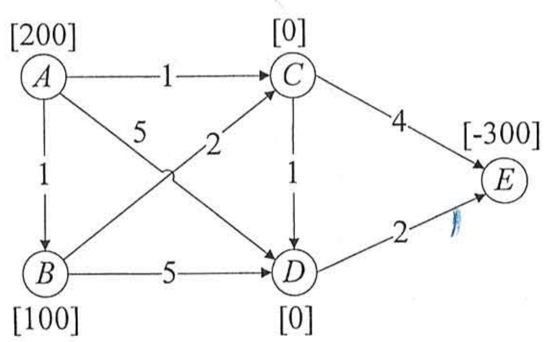
1. Consider the following linear programming problem.

$$\begin{array}{llrrrll} \text{Minimize} & z = & 2x_1 & +3x_2 & +4x_3 & & \\ \text{subject to} & & x_1 & +2x_2 & +x_3 & \geq & 3 \\ & & 2x_1 & -x_2 & +3x_3 & \geq & 4 \\ & & x_1, & x_2, & x_3 & \geq & 0 \end{array}$$

(1) Write down the dual of this problem. **(10%)**

(2) $\{x_1, x_2\}$ is an optimal basis of this problem. Use this information and the complementary slackness conditions to find the optimal solution for the dual problem. **(10%)**

2. Consider the minimum cost flow problem shown below. The net flow values are given by the nodes, the costs per unit flow are given by the arcs. The arc capacities of *(A, C)* and *(B, C)* are 100 and 150 respectively.



(1) Suppose that node capacities of node *C* and node *D* are 250 and 150 respectively. Formulate the resulting new problem as a minimum cost flow problem by drawing a network diagram. **(10%)**

(2) Solve the new problem by the network simplex method. **(20%)**

3. Consider the following nonlinear programming problem:

Maximize $f(x) = \dfrac{x_1}{x_2 + 1}$,

Subject to $x_1 - x_2 \leq 2$, and $x_1 \geq 0$, $x_2 \geq 0$.

(1) Use the KKT conditions to demonstrate that $(x_1, x_2) = (4, 2)$ is not optimal. **(10 %)**

(2) Derive a solution that does satisfy the KKT conditions. **(10 %)**

4. Every time that the team wins a game, it wins its next game with probability 0.8; every time it loses a game, it wins its next game with probability 0.3. If the team wins a game, then it has dinner together with probability 0.7, whereas if the team loses then it has dinner together with probability 0.2. What proportion of games result in a team dinner? **(15 %)**

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5. A company needs the following number of workers during each of the next five months: year 1, 15; year 2, 30; year 3, 10; year 4, 30; year 5, 20. At present, the company has 20 workers. Each worker is paid $30,000 per year. At the beginning of each year, workers may be hired or fired. It costs $10,000 to hire a worker and $20,000 to fire a worker. A newly hired worker can be used to meet the current year's worker requirement. During each year, 10% of all workers quit (workers who quit do not incur any firing cost). With dynamic programming, formulate a recursion that can be used to minimize the total cost incurred in meeting the worker requirements of the next five years. **(15 %)**