國立臺灣科技大學103學年度碩士班招生試題

系所組別： 資訊工程系碩士班
科　　目： 計算機數學

（總分為100分）

**Total Score: 100**

1. ( 15%) Let $\mathbf{v}_1 = (4, 6, 7)^\top, \mathbf{v}_2 = (0, 1, 1)^\top, \mathbf{v}_3 = (1, 1, 2)^\top$, and let $\mathbf{u}_1 = (1, 1, 1)^\top$, $\mathbf{u}_2 = (1, 2, 2)^\top, \mathbf{u}_3 = (2, 3, 4)^\top$.

   (a) Find the transition matrix from $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ to $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$. (10%)

   (b) If $\mathbf{x} = 2\mathbf{v}_1 + \mathbf{v}_2 - \mathbf{v}_3$, determine the coordinate vector of $\mathbf{x}$ with respect to $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$. (5%)

2. (15%) Let

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 2 & 0 & 1 & 1 \end{bmatrix}, \ \mathbf{c} = \begin{bmatrix} 2 \\ 7 \\ 13 \end{bmatrix}$$

   (a) What is the distance between the vector $\mathbf{b}$ and the *row space* of $A$. (10%)

   (b) What is the distance between the vector $\mathbf{c}$ and the *column space* of $A$. (5%)

3. (20%) Answer each of the statements that follow as *true* or *false*. You have to prove it if your answer is *true*, otherwise you need to give a counterexample for it. *Please note that you will* **NOT** *get any point if you only answer* true *or* false.

   (a) Let $A$ be an $m \times n$ matrix with $m > n$. Suppose that the *rank* of $A$ is $n$ then $A\mathbf{x} = \mathbf{b}$ is consistent for any $\mathbf{b} \in R^m$. (5%)

   (b) Let $Q$ be an $n \times n$ *orthogonal* matrix. Then $\|Q\mathbf{x}\|_2 = \|\mathbf{x}\|_2$. (5%)

   (c) Let $A$ and $B$ be *lower triangular* matrix and *upper triangular* matrix with the same diagonal and $A$ is *invertible*. Then $C = AB$ is *invertible*. (5%)

   (d) If $\lambda$ is an eigenvalue of a matrix $A$, then $\lambda^k$ is an eigenvalue of the matrix $A^k$, where $k$ is a positive integer. (5%)

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4. (15%) **(a)** Prove that a complete graph $K_n$, where $n \geq 3$ being a prime number, can have its edges partitioned into $\frac{1}{2}(n-1)$ disjoint Hamilton circuits. (10%)

**(b)** During a summer camp, if a group of 19 students have dinner together at a circular table everyday, and if each night each student must sit next to a completely different pair of students, then how many days can the summer camp last? (5%)

5. (10%) Let $n = 37 \times 73$. What is the probability that a randomly selected positive integer $m$, where $m < n$, is relatively prime to $n$?

6. (13%) A robot is trying to enter a room through a door that opens at random. Suppose the door opens two-thirds of the time. Furthermore, the robot is able to correctly sense an opened door with probability 0.9, and correctly sense a closed door with probability 0.7. What is the probability that the door is actually open when the robot senses an opened door?

7. (12%) Prove that an edge with the smallest weight in a weighted graph $G$ must be included in every minimum spanning tree of $G$.