國立臺灣科技大學103學年度碩士班招生試題

系所組別： 電機工程系碩士班己二組

科　　目： 電磁學

（總分為100分）

Problem 1 (total 15 points)

Non-uniform charge is distributed with a spatial dependence of ρ_0/r^2 (C/m^3) in the region of $r_0 < r < 1.5r_0$ in spherical coordinate.

(a) Find the electric flux density **D** everywhere. (10%)

(b) Evaluate the potential at $r = 1.5r_0$ if the voltage at infinite is assumed to be zero. The medium is free space. (5%)

Problem 2 (total 30 points)

Shown in Fig. 1, the region x < 0 is filled with Medium 1 while the region x > 0 is filled with Medium 2. Both media extend to infinite. Assume a uniform plane wave having electric field of

$$\mathbf{E}_i = E_0 \sin(10^9 \pi t - 5\pi x)\mathbf{a}_z + E_0 \cos(10^9 \pi t - 5\pi x)\mathbf{a}_y \quad \text{(V/m)}$$

is normally incident from Medium 1 onto the interface $x = 0$. Please answer the following questions:

(a) Determine the linear frequency, phase constant, phase velocity of the incident wave in Medium 1. (5%)

(b) Determine the relative dielectric constant of Medium 1. (5%)

(c) Determine the polarization sense of the incident wave. (5%)

(d) Determine the reflection and transmission coefficients at the interface. (5%)

(e) Express the transmitted wave magnetic field **H** in Medium 2 in phasor form. (5%)

(f) Calculate the time-average power delivered by the incident wave in a square meter area parallel to the yz-plane ($x < 0$) (5%)

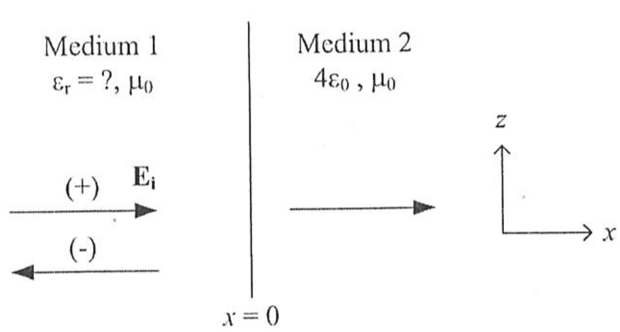

Fig. 1

Problem 3 (total 5 points)

Given a vector field **B**

$$\mathbf{B} = (mxy + nxz^2)\mathbf{a}_x + y^2\mathbf{a}_y - z^3\mathbf{a}_z,$$

find m and n such that **B** can represent a magnetic field in rectangular coordinate system with $|x|, |y|, |z| \le 1$.

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Problem 4 (total 14 points)

For a lossless transmission line system shown in Fig. 2, if a wave carrying 1 Watt of power is incident from the left of Line 1, find

(a) The power reflected back to Line 1; (4%)

(b) The power transmitted to the load resistor of Line 2; (4%)

(c) The power dissipated on the resistor at the junction (2%)

(d) If the load resistance of Line 3 is removed and the Line 3 length is a quarter wavelengths, what is the power reflected back to Line 1. (4%)

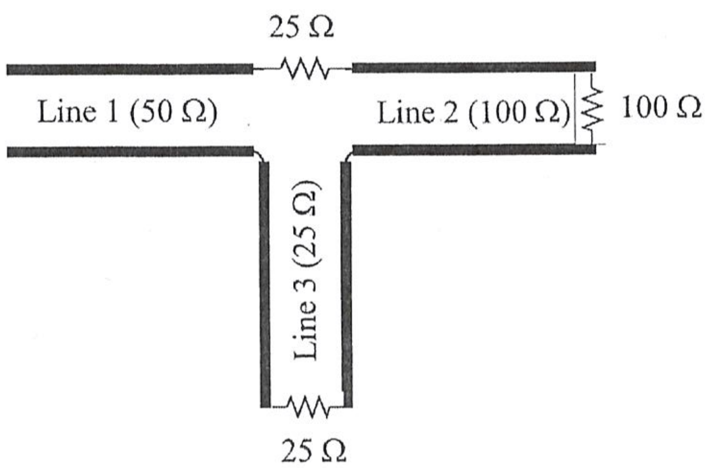

Fig. 2

Problem 5 (total 12 points)

Using following definitions and relationships, find

(a) the gradient of $r^2\cos\theta$ in spherical coordinate. (2%)

(b) the Laplacian of (a) in spherical coordinate. (10%)

$$\nabla f = \frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right)\hat{\boldsymbol{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\hat{\boldsymbol{\phi}}$$

$$\begin{aligned}\nabla^2\mathbf{A} =& \left(\Delta A_r - \frac{2A_r}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial (A_\theta\sin\theta)}{\partial\theta} - \frac{2}{r^2\sin\theta}\frac{\partial A_\phi}{\partial\phi}\right)\hat{\boldsymbol{r}} \\ &+ \left(\Delta A_\theta - \frac{A_\theta}{r^2\sin^2\theta} + \frac{2}{r^2}\frac{\partial A_r}{\partial\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\phi}{\partial\phi}\right)\hat{\boldsymbol{\theta}} \\ &+ \left(\Delta A_\phi - \frac{A_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial A_r}{\partial\phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial\phi}\right)\hat{\boldsymbol{\phi}}\end{aligned}$$

$$\Delta f \equiv \nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

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Problem 6 (total 24 points)

For points A~F on impedance Smith Chart shown in the next page, assuming the line characteristic impedance is 75 Ω:

(a) For Point A, find its impedance Z_A and admittance Y_A. (4%)

(b) For Point B, find the magnitude and phase in degree of its reflection coefficient. (4%)

(c) For Point C, find its SWR value. (2%)

(d) For Point D, to implement a quarter wavelengths transformer, what is the minimum line length needed before inserting the transformer and what is the characteristic impedance of the transformer? Note the transformer characteristic impedance cannot be greater than 75 Ω. (6%)

(e) Point E is located on the admittance chart. The matching can be achieved by connecting a short-ended stub in parallel. What is the short stub length? (4%)

(f) Point F is located on the impedance chart. The matching can be achieved by connecting an open-ended stub in series. What is the open stub length? (4%)

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