國立臺灣科技大學103學年度碩士班招生試題

系所組別： 電子工程系碩士班乙三組、丙組

科　　目： 電磁學

（總分為100分）

1. Prove the divergence in the various coordinate systems:

(a) $\nabla \bullet \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial}{\partial \varphi}(A_\varphi) + \frac{\partial A_z}{\partial z}$ in cylindrical coordinates (10%)

(b) $\nabla \bullet \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$ in spherical coordinates (10%)

[Hint: $\nabla = \hat{r}\frac{\partial}{\partial r} + \hat{\varphi}\frac{1}{r}\frac{\partial}{\partial \varphi} + \hat{z}\frac{\partial}{\partial z}$ and $\nabla = \hat{r}\frac{\partial}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial}{\partial \theta} + \frac{\hat{\varphi}}{r\sin\theta}\frac{\partial}{\partial \varphi}$]

2. (a) Calculate the electric field due to a thin ring of charge having a uniform charge density σ for a point p on the z axis, which is through the thin ring center. ρ and Δρ are the thin ring radius and thickness, respectively. The p point in Cartesian coordinates can be assumed as (0,0,z), as shown in **Fig. 1**. (10%)

(b) Using the result of 2 (a), calculate the electrostatic filed for a uniformly charged plane sheet of charge density σ for a point p sufficiently near the plane, as shown in **Fig. 2**. The definitions for ρ, Δρ and z in **Fig. 2** are the same as 2 (a). By "sufficiently near" we mean that we may consider it to be a good approximation to let the plane become infinite in extent. (10%)

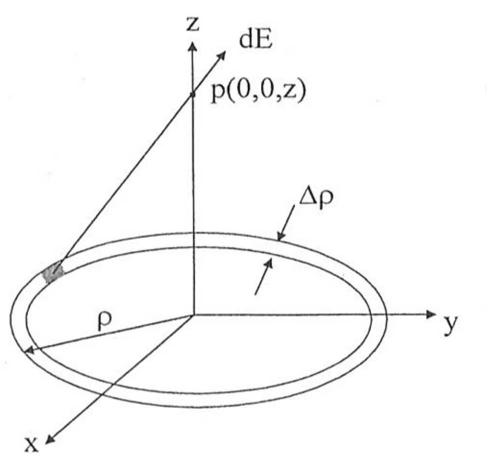


Fig. 1 The electric filed at the axis of a ring of charge

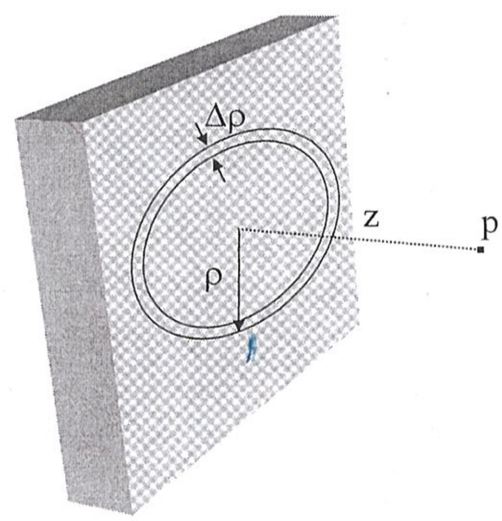


Fig. 2 The electric field of an infinite surface charge distribution by integrating the field produced by a ring of charge

3. A conducting material of uniform thickness *h* and conductivity σ has the shape of a quarter of a flat circular washer, with inner radius a and outer radius b, as shown in **Fig. 3**. Determine the resistance between the end faces. (10%)

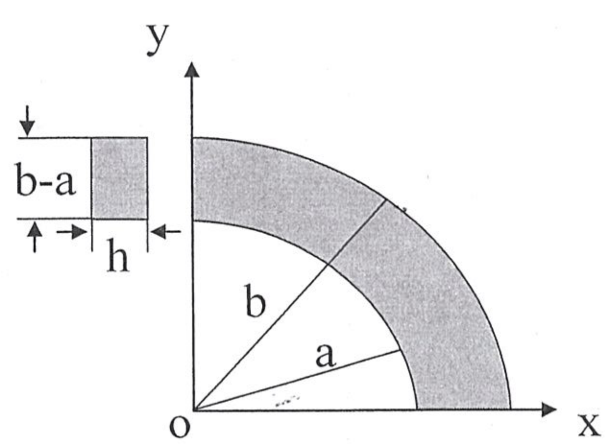
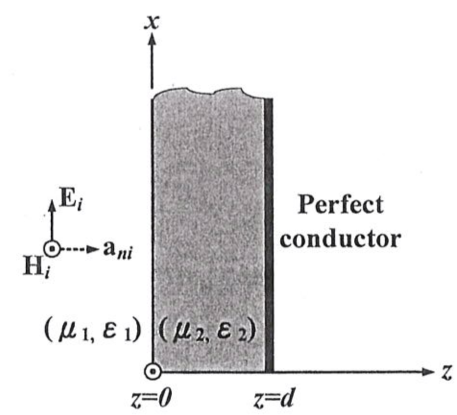


Fig. 3 A square of a flat conducting circular washer

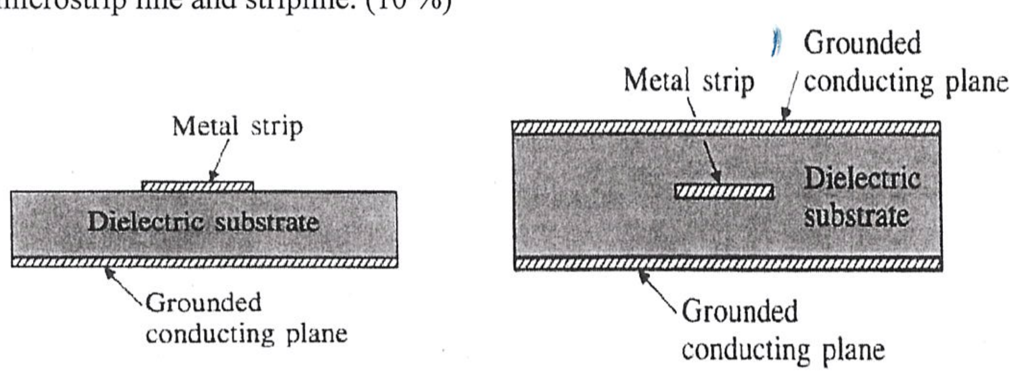
國立臺灣科技大學103學年度碩士班招生試題
系所組別： 電子工程系碩士班乙三組、丙組
科　　目： 電磁學

（總分為100分）

4. A uniform plane wave in medium 1 (μ_1, ε_1) is incident normally onto a lossless dielectric slab (μ_2, ε_2) of thickness d backed by a perfectly conducting plane as shown below. (a) Please calculate the input impedance seen by the uniform plane wave in medium 1. (10%) (b) Please calculate the reflection coefficient seen by the uniform plane wave in medium 1. (10%)



5. Consider the cross-sections of the microstrip line and stripline shown below. Assume that both structures have the same dimensions except that the substrate thickness of the stripline is twice that of the microstrip line. Assuming that the width of the metal strip is very wide, the stripline can be viewed as two microstrip lines in parallel. (a) What is the relationship between the propagation constants of the microstrip line and stripline? (10%) (b) What is the relationship between the characteristic impedances of the microstrip line and stripline? (10 %)



6. Two infinite lines are placed in parallel to the y axis and separated by a distance d. Assume that both lines are applied with a current I in the $+y$ direction. What should the values of the distance d be (in terms of wavelength) that will make the radiation field zero in the $+z$ axis? (10%)