

國立臺灣科技大學103學年度碩士班招生試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組

科目：工程數學

(總分為100分)

1. Solve the initial-value problem: $y' \cos x + (3y-1)\sec x = 0$, $y(\pi/4) = 4/3$. (20%)

2. (a) Find the Laplace transform of $te^{-t} \cos(3t)$. (6%)

(b) Find the Laplace transform of $f(t) = \begin{cases} 2t - \sin(t) & \text{for } 0 \leq t < \pi \\ 0 & \text{for } t \geq \pi \end{cases}$. (7%)

(c) Find the inverse Laplace transform of $\frac{4e^{-3s}}{s^2 + 4s + 20}$. (7%)

3. (a) Compute the eigenvalues of matrix $\mathbf{A} = \begin{bmatrix} 2 & -3 & -1 \\ -5 & 4 & 2 \\ 1 & -5 & -1 \end{bmatrix}$. (8%)

(b) The eigenvalues of matrix $\mathbf{B} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ are 3, 2 and 0. Find a matrix \mathbf{P} such

that $\mathbf{P}'\mathbf{B}\mathbf{P} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, where \mathbf{P}' is the transpose of matrix \mathbf{P} . (12%)

4. Verify Stokes's theorem, by calculating both sides of the equation, for the case $\vec{v} = xz\vec{j}$, and S is the surface $z = 4 - y^2$ cut off by the planes $x = 0$, $z = 0$ and $y = x$. (20%)

5. Solve in detail the boundary-value problem: $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial t^2}$, $t \geq 0$

with $w(0, y, t) = w(2, y, t) = 0$, $\frac{\partial w}{\partial y}(x, 0, t) = \frac{\partial w}{\partial y}(x, 1, t) = 0$,

$w(x, y, 0) = (x^2 - 2x)\sin^2 \pi y$, $\frac{\partial w}{\partial t}(x, y, 0) = 0$. (20%)

