

國立臺灣科技大學103學年度碩士班招生試題

系所組別： 自動化及控制研究所碩士班甲組、乙組

科 目： 工程數學

(總分為100分)

1. Solve the following differential equations:

(1) $xy' + 2y = 4y^2$ (10%)

(2) $x^2y'' - 2xy' + 2y = 2\ln(x) - 1$ (10%)

2. Use Laplace Transform method to solve the following equations.

(15%)

$$x'' - 2x' + 3y' + 2y = 4$$

$$-x' + 2y' + 3y = 0$$

where $x(0) = x'(0) = y(0) = 0$

3. Find the first five nonzero terms of the Maclaurin power series solution of the initial value problem. (15%)

$$y'' + 3y' + 2y = x \quad \text{where} \quad y(0) = 0 \quad \text{and} \quad y'(0) = 1$$



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4. Consider the matrix A .

$$A = \begin{pmatrix} -1 & 0 \\ 1 & -5 \end{pmatrix}; \quad (15\%)$$

(1) Find the Eigenvalues and Eigenvectors. (8%)

(2) Compute A^{18} . (7%)

5. Use the Fourier transform to solve

$$y'' + 6y' + 5y = \delta(t-3). \quad (15\%)$$

6. Solve the telegraph equation

$$\frac{\partial^2 u}{\partial t^2} + A \frac{\partial u}{\partial t} + Bu = c^2 \frac{\partial^2 u}{\partial x^2}$$

for $0 < x < L$, $t > 0$. A and B are positive constants. The boundary conditions are

$$u(0, t) = u(L, t) = 0 \quad \text{for } t \geq 0,$$

and the initial conditions are

$$u(x, 0) = f(x), \quad \frac{\partial u}{\partial t}(x, 0) = 0 \quad \text{for } 0 \leq x \leq L.$$

Assume that $A^2 L^2 < 4(BL^2 + c^2 \pi^2)$. (20%)