國立臺北科技大學 103 學年度碩士班招生考試

系所組別：3510 化學工程與生物科技系化學工程碩士班甲組

第三節 工程數學 試題

注意事項：

1. 本試題共 9 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Compute the convolution of e^{5t} and e^{8t}. (10%)

2. Find the Laplace transform of $(t-2)^2 u(t-1)$. (10%)

3. If **A** is a 3×3 real symmetric matrix , **I** is the identity matrix, the eigenvalues of **A** are $\lambda_1 = 1$, $\lambda_2 = 1$, and $\lambda_3 = -1$. Find $\mathbf{A}^{2000}$. (10%)

4. Find the orthogonal trajectories of the family of curves
   $y = C \sin(x)$. (10%)

5. Find a vector field whose curl is $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ or prove that no such vector field exists. (10%)

6. A force field f in 3-space is given by $f(x,y,z) = x\mathbf{i}+y\mathbf{j}+(xz-y)\mathbf{k}$. Compute the work done by this force in moving a particle from (0,0,0) to (1,2,4) along the line segment joining these two points. (10%)

7. Find the surface area of the part of the graph of the paraboloid $z = x^2+y^2$ between the plane $z = 1$ and $z = 4$. (15%)

8. Find the Fourier series for the function $f(x) = x^3-2x^2$, defined on $-\pi < x < \pi$. (15%)

9. The solution to the problem

   $$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad \text{for } 0 < x < \pi , t > 0$$

$$\frac{\partial u}{\partial x}(0,t) = 0, \quad \frac{\partial u}{\partial x}(\pi,t) = 0$$

$$u(x,0) = (x-\frac{\pi}{2})^2$$

is $u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(nx)e^{-n^2 t}$. What is the value of A_6 ?   (10%)