國立臺北科技大學 103 學年度碩士班招生考試

系所組別：2151 電機工程系碩士班戊組

第三節　線性代數　試題（選考）

注意事項：

1. 本試題共 3 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Let $T=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$.

(10%) (a) Diagonalize the matrix T, if possible.

(5%) (b) If the transformation $T:\mathbb{R}^2\to\mathbb{R}^2$, find a basis B for the domain of T such that the matrix of T relative to B is diagonal.

2. Let $T=\begin{bmatrix}1 & 2 & -1\\ -3 & -5 & 0\\ 4 & 6 & 1\end{bmatrix}$ be the change-of-coordinate matrix from a basis U to a basis V.

(5%) (a) Determine if this coordinate transformation maps $\mathbb{R}^3$ onto $\mathbb{R}^3$, and justify your answer.

(10%) (b) Given the basis $V=\left\{[-2\quad 2\quad 3]^T,[-8\quad 5\quad 2]^T,[-7\quad 2\quad 6]^T\right\}$, find a basis U.

(10%) (c) Find the change-of-coordinate matrix from a basis V to a basis U, if possible.

3. Let $A=\begin{bmatrix}1 & 3 & 5\\ 1 & 1 & 0\\ 1 & 1 & 2\\ 1 & 3 & 3\end{bmatrix}$.

(5%) (a) Determine the rank of A.

(5%) (b) Determine the kernel of A.

(5%) (c) Determine if the linear transformation $\mathbf{x}\mapsto A\mathbf{x}$ for any vector $\mathbf{x}\in\mathbb{R}^3$ is one-to-one, and justify your answer.

(15%) (d) Find an orthogonal basis for the column space of A.

(15%) (e) Find a QR factorization of A.

(15%) (f) Given the data (3,5,3), (1,0,5), (1,2,7), (3,3,-3) for the variables (u, v, y). Find the model with equation $y=a_1+a_2u+a_3v$ for predicting y from u and v that gives a least-squares fit to such data.