國立臺北科技大學 103 學年度碩士班招生考試

系所組別：2140 電機工程系碩士班丁組

第三節　工程數學　試題

注意事項：

1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. The joint probability density function (pdf) of random variables X and Y is

$$f_{XY}(x,y)=\begin{cases} c & 0\le x\le 1, 0\le y\le 3 \\ 0 & \text{else} \end{cases}$$

(a) (5%) What is the value of constant c?

(b) (5%) Find the pdf of random variable X.

(c) (5%) Find the joint cumulative distribution function (cdf) $F_{XY}(x,y)$ of random variables X and Y.

(d) (5%) Are X and Y independent? Explain your answer?

2. X and Y are discrete random variables with joint probability mass function (pmf)

$$p_{XY}(x,y)=\begin{cases} (1/9)(2/3)^{x+y-2} & x,y\in\{1,2,3,\cdots\} \\ 0 & \text{else} \end{cases}.$$

(a) (5%) Find the pmf of random variable $Z=X+Y$.

(b) (5%) Find the probability of the event $\{X \text{ is odd}\}$.

3. Let $X_1, X_2, \cdots$ be independent random variables with common pdf

$$f_X(x)=\begin{cases} e^{-x} & x\ge 0 \\ 0 & x<0 \end{cases}.$$ Let $Y_n=\max\{X_1,\cdots,X_n\}$.

(a) (4%) Show that the moment generating function of X_1 is $M_{X_1}(s)=E[e^{sX_1}]=\frac{1}{1-s}$ for $s<1$.

(b) (8%) Find EX_1 and $\mathrm{Var}(X_1)$.

(c) (4%) For $a>0$, Find $P(Y_n>a)$.

(d) (4%) For $b>a>0$, Find $P(Y_n\le a, Y_{n+1}>b)$.

4.

$$\text{Let } A = \begin{bmatrix} 1 & 1/3 & -2/3 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 \\ 0 & -1/6 & 1/3 & 0 & 0 \\ -1/2 & -2/3 & 1/3 & 1/2 & 0 \\ 0 & -1/3 & 0 & 0 & 1/3 \end{bmatrix} \text{ and } b = \begin{bmatrix} 5 \\ 5 \\ 1 \\ 7 \\ 8 \end{bmatrix};$$

(a) (5%) Solve for x such that $Ax = b$;

(b) (5%) Find all eigenvalues of A, and list them in descending order;

(c) (5%) Find a 5×5 matrix X and X^{-1} such that $X^{-1}AX = D$, where D is the diagonal matrix whose diagonal elements are eigenvalues of A in descending order;

(d) (10%) Find the determinant of A and the inverse of A.

(e) (5%) Find e^A.

5. (10%) Let L be the linear mapping from R^3 to R^3 defined by $L(x) = Ax$, where

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ 7 & -6 & 3 \end{bmatrix}, \text{ and let } v_1 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}.$$

Find the transition matrix corresponding to a change of basis from $[e_1, e_2, e_3]$ to $[v_1, v_2, v_3]$, and use it to determine the matrix B representing L with respect to $[v_1, v_2, v_3]$.

6. (10%)

$$\text{Let } A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \\ 5 & -2 \\ -6 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 7 \\ -7 & 2 \\ 4 & 5 \\ 2 & -1 \end{bmatrix};$$

Find AB^T, A^TB and the inner product $\langle A, B \rangle$.