注意事項：

1. 本試題共五題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

---

1. Solve the following differential equations

(1) $y''' - 3y' + 2y = 8e^x + 4x + 1$　(15%)

(2) $y' + 2y = x^2 y^3$　(15%)

2. (1) Evaluate the integral $I = 3\int_0^\infty \sqrt{x}e^{-x^3}dx$　(10%)

(2) Given that Laplace transform $L\{t^{-1/2}\} = (\pi/s)^{1/2}$, please find the value of $L\{t^{1/2}\}$. (15%)

3. The equation $\begin{cases} 3x - y = 4 \\ x + 2y = 0 \\ 2x + y = 1 \end{cases}$ have no solution. Find the vector $X = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ that best approximates a solution. (10%)

4. Please prove the following theorem.

Let A, B, C and D with appropriate dimensions. If A, $D - CA^{-1}B$, and D are nonsingular, then $\left(A - BD^{-1}C\right)^{-1}$ is nonsingular, and

$\left(A - BD^{-1}C\right)^{-1} = A^{-1} + A^{-1}B\left(D - CA^{-1}B\right)CA^{-1}$. (20%)

5. Let $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$.

(1) Find the eigenvalues? (5%)

(2) The corresponding orthonormal eigenvectors of A. (10%)