國立臺北科技大學 103 學年度碩士班招生考試

系所組別：1310、1320、1330 車輛工程系碩士班甲、乙、丙組

第三節　工程數學　試題

注意事項：

1. 本試題共 7 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (10%) Find the general solution for $2xy^2+2xy+(x^2y+x^2)y'=0$.

2. (10%) Solve the initial value problem $y'+\frac{1}{x-2}y=3x$; $y(3)=4$.

3. (15%) A system has its equation and initial values as: $y''-4y'+3y=10\cos x$;
   $y(0)=1; y'(0)=-1$.
   (1) Solve this initial value problem; (10%)
   (2) The transient output of this system contains 2 terms, $y_1(t)$ and $y_2(t)$. Draw the graphs for $y_1(t)$ and $y_2(t)$ separately. (5%)

4. (15%) Let $f(t)=4e^{3t}+3\sin(2t)$, find the Laplace transform of function $f(t)$ by the definition of Laplace Transform. (No points obtained if the definition is not used or the calculation process is not expressed clearly)

5. (15%) For two matrices $A=[a_{ij}]_{3\times3}$ and $B=[b_{ij}]_{3\times3}$, prove the following equations by the terms of a_{11}, a_{12}, ...a_{33} and b_{11}, b_{12}, ...b_{33}.
   (1) $2A+3B=3B+2A$; (10%)
   (2) (A+B)+C = A+(B+C). (5%)

6. (15%) Two matrices $A=\begin{bmatrix} 2 & -1 & 3 \\ -2 & 1 & 4 \\ 1 & 2 & -2 \end{bmatrix}$ and $B=\begin{bmatrix} -3 & 4 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$, find:

(1) $AB+(A+B)$ (5%)

(2) $B^T+A^T+B^TA^T$ (5%)

(3) $B^{-1}A^{-1}$ (5%)

7. (20%) For a matrix A, (1) find a basis of eigenvectors of matrix A (10%); (2) diagonalize the matrix A (5%); (3) find the value of A^6 (5%)

$$A=\begin{bmatrix} -1 & 2 & -2 \\ 2 & 4 & 1 \\ 2 & 1 & 4 \end{bmatrix}$$