國立臺灣師範大學 103 學年度碩士班招生考試試題

科目：微積分　　　　　　　　　　　　　適用系所：全球經營與策略研究所

注意：1.本試題共 2　頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

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1. A retailer's weekly profit from the sale of two brands of coffee is

   $$P(x, y) = (x - 40)(60 - 4x + 5y) + (y - 50)(70 + 5x - 7y)$$

   cents, where x is the price per bottle of the first brand and y is the price per bottle of the second, both in cents. Currently the first brand sells for 80 cents per bottle and the second, for 83 cents per bottle. Use calculus to estimate the change in weekly profit that will result if the retailer decides to raise the price of the first brand by 1 cent and the price of the second by 2 cents. (10 points)

2. At a factory, the monthly output is $Q = 80K^{\frac{1}{2}}L^{\frac{1}{4}}$ units, where K denotes the capital investment measured in units of \$1,000 and L the size of the labor force measured in worker-hours. The current capital investment is \$900,000 and 10,000 worker-hours of labor are used each month. Estimate the change in output that will result if capital investment is increased by \$2,000 and labor is increased by 1 worker-hour. (10 points)

3. John is constructing an ornate museum display box from material that costs \$1 per square inch for the bottom, \$3 per square inch for the sides, and \$7 per square inch for the top. If the total volume is to be 36 in.^{2}, what dimensions should John choose to minimize the total cost of construction? What is the minimal cost of construction? (10 points)

4. Find the area of the region C bounded by the curves $y = x^3$ and $y = x^2$. (10 points)

5. Find local maxima and local minima of the function $y = x^3 - 3x + 4$ on the interval $-\infty < x < \infty$. (10 points)

6. The marketing manager of a company in Taipei finds that his profit varies periodically throughout the year and at week t is $P(t)$ thousand dollars, where $P(t) = 19 - 10\sin\left[\dfrac{\pi}{52}(2 - t)\right]$ for $1 \le t \le 52$. Find the average weekly profit over the year. (10 points)

7. Let X be a random variable that measures the duration of phone calls in a company, and assume that X has an exponential distribution with density function

$$f(t) = \begin{cases} 0.5e^{-0.5t} & \text{if } t \geq 0 \\ 0 & \text{if } t \geq 0 \end{cases}$$

where t denotes the duration (in minutes) of a randomly selected call.

(a) Find the probability that a randomly selected call lasts no more than 2 minutes. (10 points)

(b) Find the probability that a randomly selected call will last at least 1 minute. (10 points)

(c) How long would you expect a randomly selected call to last? (10 points)

8. The population of a country grows logistically in such a way that after t years, the population is P million, where

$$P(t) = \frac{1}{1 + e^{-1.5t}}$$

Thus, the average population over the next 10 years is given by the integral

$$P_{ave} = \frac{1}{10} \int_0^{10} P(t)dt$$

Estimate the value of P_{ave} by integrating the Taylor polynomial of degree 2 for $P(t)$ expanded at t=0. (10 points)