

國立臺灣師範大學 103 學年度碩士班招生考試試題

科目：工程數學（能源應用組）

適用系所：工業教育學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. Prove the Laplace transform as follows: (共 15 分)

(a) $L[e^{\alpha} f(t)] = F(s - \alpha)$, where α is a constant. (7 分)

(b) $L[u(t - \alpha)f(t - \alpha)] = e^{-\alpha} F(s)$, where α is a constant, $u(t - \alpha) = 1$ if $t \geq \alpha$;
 $u(t - \alpha) = 0$ if $t < \alpha$ (8 分)

(Hint: $L[f(t)] = \int_0^{\infty} e^{-st} f(t) dt$)

2. Solve the problems below: (共 15 分)

(a) the Laplace transform of $f(t) = e^{2t} \int_0^t (u + \cos 3u + u^3) du$ (8 分)

(b) the inverse Laplace transform of $F(s) = \frac{5s}{s^2 + s - 6}$ (7 分)

3. Derive the general solutions of two equations as follows: (共 15 分)

(a) $(D - 1)(D - 2)^2(D + 2D + 5)^3 y = 0$, where $y = y(x)$, $D \equiv d/dx$. (7 分)

(b) $y'' - y' + y = \sin 2x$ (8 分)

4. Please prove: if $A(x)$ and $B(x)$ are two functions of x , then for the first-order ordinary differential equation $y' + A(x)y = B(x)$: (共 10 分)

(a) the integrating factor $I(x) = e^{\int A(x) dx}$ (5 分)

(b) the general solution is $y = I(x)^{-1} \int I(x) B(x) dx + C$, where C is a constant (5 分)

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5. Please solve the general solutions of two first-order differential equations as follows: (共 15 分)

(a) $2(x + y)dx - (3x + 3y - 5)dy = 0$ (8 分)

(b) $xdy - ydx - x^2 \sin y dy = 0$ (7 分)

6. Please calculate the inverse of: (共 10 分)

(a) $A_1 = \begin{bmatrix} 5 & 1 \\ 3 & 2 \end{bmatrix}$ (5 分) and (b) $A_2 = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 4 \\ 1 & 3 & 7 \end{bmatrix}$ (5 分)

7. Find all the fourth roots of 16 (10 分)

8. Please solve the general solution of the Euler-Cauchy Equation as follows: (10 分)

$x^2 y'' - xy' + y = 0$ ($x > 0$) (Hint: let $x = e^t$ and transfer the x function to t function)