

# 國立臺灣師範大學 103 學年度碩士班招生考試試題

科目：工程數學（電機電子組）

適用系所：工業教育學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. Let  $A = \begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$ . Find  $A^{50}$ . (10 分)

2. Let  $T(a, b, c, d) = \langle 8d - a - b - 4c, c + a - b \rangle$ . Determine whether the given function  $T$  is a linear transformation. Justify your answer. (15 分)

3. Determine a matrix  $P$  that diagonalizes  $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & -2 & 4 \end{bmatrix}$ . (10 分)

4. Solve the equation  $y(t) = \frac{1}{2} + \int_0^t y(t-\tau)e^{-3\tau} d\tau$ . (10 分)

5. Find the Fourier series of the function  $f(x)$  of period  $2\pi$  (10 分)

$$f(x) = 2|x|, \quad |x| < \pi$$

6. Find the general solution of the system (15 分)

$$\begin{cases} \frac{dx}{dt} = 3x - y + z \\ \frac{dy}{dt} = x + y - z \\ \frac{dz}{dt} = x - y + z \end{cases}, \quad x(0) = 2, y(0) = 1, z(0) = 3.$$

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7. Find the general solution of the differential equation (10 分)

$$\frac{dy}{dx} + \frac{xy}{1+x^2} = \frac{\sqrt{1+x^2}}{1+x^2}$$

8. Solve the differential equation  $\frac{dy}{dx} = 3xy^4 - xy$ . (10 分)

9. Solve the differential equation  $y'' - 2y' + y = 4x + 27\sin(3x)$ . (10 分)