國立臺灣師範大學 103 學年度碩士班招生考試試題

科目：工程數學（光機電系統組）　　　　適用系所：機電工程學系

注意：1.本試題共 3 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. Solve the solution y(t) of the following differential equation: (10 分)

$$y'' + 2y' + 10y = e^{-t}, \quad y(0) = 0, y'(0) = 0$$

2. Solve the general solution of the following system: (15 分)

$$x' + y' = x$$
$$x' + 2y' = -x - 2y$$

3. Find the Laplace transform of the function f(t) defined by. (10 分)

$$f(t) = 0, \quad \text{for } t < 0$$
$$= t^2 \sin \omega t, \quad \text{for } t \geqq 0$$

4. Find and plot the output response y(t) to the unit step input r(t)=μ(t) for the following transfer function of a system is

$$\frac{Y(s)}{R(s)} = \frac{100}{S^2 + 100}$$. (15 分)

5. Consider a dynamic system represented by $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$ and $y(t) = \mathbf{C}\mathbf{x}(t)$, in which $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $\mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}$. Here, $\mathbf{x}(t)$, $u(t)$ and $y(t)$ denote the state vector, input and output, respectively. (25 分)

   (a) Find the transfer function of the system from input to output using $\frac{Y(s)}{U(s)} = \mathbf{C}[s\mathbf{I} - \mathbf{A}]^{-1}\mathbf{B}$, in which $Y(s)$ and $U(s)$ denote the Laplace transforms of $u(t)$ and $y(t)$, respectively. (5 分)

(b) Given $\mathbf{x}(0)=\begin{bmatrix}1 & 0\end{bmatrix}^{\mathrm{T}}$, find the zero-input state response $\mathbf{x}(t)$ of the system by trying a vector solution of the form: $\mathbf{x}(t)=\xi e^{\lambda t}$, in which ξ is a constant vector and λ is a constant scalar. Here, the zero-input state response is the state response with $u(t)=0$ for $t\geq 0$. *Note:* This is a typical application of matrix eigenvalue problems. (10 分)

(c) Given $\mathbf{x}(0)=\begin{bmatrix}1 & 0\end{bmatrix}^{\mathrm{T}}$, find the zero-input state response $\mathbf{x}(t)$ of the system by assuming a vector solution of the form: $\mathbf{x}(t)=e^{\mathbf{A}t}\mathbf{c}$, in which $\mathbf{c}$ is a constant vector. *Note:* $e^{\mathbf{A}t}$ denotes the matrix exponential of $\mathbf{A}t$. (10 分)

6. Consider a set of functions $\{1, \cos\theta\}$ on the interval $0\leq\theta\leq\pi$. (15 分)

(a) Is this an orthogonal set with respect to the weight function $r(\theta)=\sin\theta$? Give a detailed explanation. (5 分)

(b) Define a function $f(\theta)=3\cos^2\theta-1$. Let $g(\theta)=\alpha_1+\alpha_2\cos\theta$, in which α_1 and α_2 are constants. Please determine the values of α_1 and α_2 so that $\int_0^{\pi} r(\theta)(f(\theta)-g(\theta))^2\,d\theta$ is minimized. (10 分)

7. Consider a scalar function $f(x,y,z)=x^3+y^5-z^4$. (10 分)

(a) Find the gradient of $f(x,y,z)$. (5 分)

(b) Evaluate the line integral $\int_C \nabla f\cdot d\mathbf{r}$, in which the curve C is represented by a parametric representation: $\mathbf{r}(t)=[x(t),\quad y(t),\quad z(t)]\quad=[\cos t,\quad \sin t,\quad t]$

from (1, 0, 0) to (1, 0, 4π). (5 分)