

國立臺灣師範大學 103 學年度碩士班招生考試試題

科目：應用數學

適用系所：物理學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. Use the method of series expansion to find the solutions $y(x)$ around $x = 0$ for the following differential equation. Notice you need to find two independent solutions.

(20 points)

$$4xy'' + 2(1 - x)y' - y = 0.$$

2. Solve the following simultaneous equations with the given initial conditions. (20 points)

$$\frac{dx(t)}{dt} - 2y(t) = -\sin(t),$$

$$\frac{dy(t)}{dt} + 2x(t) = 5\cos(t),$$

$$x(0) = 3, \quad y(0) = 2.$$

3. (a) Find the Fourier series of $f(x) = |x|$ in the range $-\pi \leq x < \pi$. (10 points)

(b) By integrating the results obtained in (a) term by term from 0 to x , evaluate the following series. You can assume that $x > 0$. (10 points)

$$1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \frac{1}{9^3} - \frac{1}{11^3} + \dots$$

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4. A rectangular metal plate occupied the region $0 \leq x \leq a$ and $0 \leq y \leq b$ in the xy plane. The temperatures $u(x, y, t)$ of the boundaries of the metal plate at $x = 0, y = 0, y = b$ are fixed to zero. In addition, the temperature at the side $x = a$ of the metal plate is fixed to a constant u_0 . Find the steady-state temperature $u(x', y')$ at any site (x', y') of the metal plate. (20 points)
5. Use the residue theorem (and other relevant theorems as well) in complex analysis to evaluate the following two integrals. In the second integral n is a non-negative integer, a is real and $a > 1$. (Each integral counts 10 points)

$$\int_0^{\infty} \cos(x^2) dx,$$
$$\int_0^{\pi} \frac{\cos(nx)}{1 - 2a \cos(x) + a^2} dx.$$