國立臺灣師範大學 103 學年度碩士班招生考試試題

科目：**數學基礎** 適用系所：**資訊工程學系**

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

(1) Derive the equation of a plane in the $x-y-z$ space, which contains the point (-1, 3, 2) and is perpendicular to the vector (6, 4, -9). (5 分)

(2) Let A be an $n \times n$ nonsingular matrix. Then, the non-homogeneous system of equations $AX = B$ has a unique solution. (a) The solution can be obtained using Cramer's rule. Give the formula of Cramer's rule. (b) Apply Cramer's rule to solve the system: $x_1 - 3x_2 - 4x_3 = 1,\ -x_1 + x_2 - 3x_3 = 14,\ x_2 - 3x_3 = 5$. (c) Find the unique function $f(x) = c_1 e^x + c_2 e^{2x} + c_3 e^{3x}$ that satisfies $f(0) = 1, f'(0) = 2$ and $f''(0) = 0$. (15 分)

(3) Let A and B be $n \times n$ nonsingular matrices and $\det(A) = a \neq 0$ and $\det(B) = b \neq 0$. Answer (a) $\det(AB)$, (b) $\det((AB)^T)$, (c) $\det(\det(A)A)$, (d) $\det(\det(A)B)$, and (e) $\det(\det(B)B)/\det(\det(B)A)$. (15 分)

(4) Let $F: P \to P$ be defined by

$$F(a_0 + a_1 x + a_2 x^2) = 2(a_1 - a_2) + (2a_0 + 3a_2)x + 3a_2 x^2$$

(a) Let $A = \{1, x, x^2\}$ be a basis for *P*. Give the matrix *M* corresponding to mapping *F* with respect to *A*.

(b) Find the eigenvectors and the associated eigenvalues of *M*.

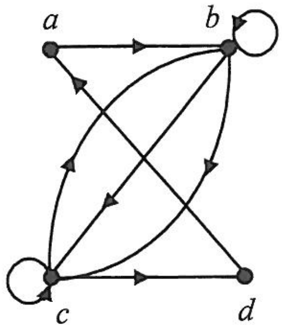
(c) Let *B* denote the basis of *P* that consists of the eigenvectors of *M*. Give the matrix M' corresponding to *F* with respect to basis *B*. (15 分)

(5) Suppose that *a* and *b* are integers, $a \equiv 4 \pmod{13}$, and $b \equiv 9 \pmod{13}$. Find the integer *c* with $0 \le c \le 12$ such that $c \equiv a^2 + b^2 \pmod{13}$. (3 分)

(6) Determine whether the relation represented by the zero–one matrix is an equivalence relation. (3 分)

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(7) Find the adjacency matrix of the given directed multigraph with respect to the vertices listed in alphabetic order. (5 分)



(8) For the logical expression Q ∧ (P ∨ ¬R),

(a) Find its disjunctive normal form.

(b) Find its conjunctive normal form. (6 分)

(9) Give the formula for the coefficient of x^k in the expansion of $(x^2 - 1/x)^{100}$, where k is an integer. (6 分)

(10) Express gcd(252, 198) = 18 as a linear combination of 252 and 198. (6 分)

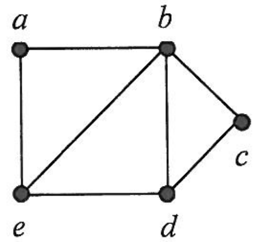
(11) The Lucus numbers satisfy the recurrence relation

$L_n = L_{n-1} + L_{n-2}$,

and the initial conditions $L_0 = 2$ and $L_1 = 1$.

Find an explicit formula for the Lucus numbers. (6 分)

(12) Determine whether the graph has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists. (6 分)



(13) Answer "Yes" or "No" for the following questions.

(a) Is the function $f(n) = n^2 + 100$ from **Z** to **Z** one-to-one?

(b) Is the function $f(x) = (x^2 + 1)/(x^2 + 2)$ from **R** to **R** a bijection?

(c) If f and $f \circ g$ are onto, does it follow that g is onto? (9 分)