

淡江大學 103 學年度碩士班招生考試試題

18

系別：數學學系

科目：基礎數學（含微積分、線性代數）

考試日期：3月2日(星期日) 第2節

本試題共 8 大題， 1 頁

一、(20%) Find the integrals :

(1) $\int_0^1 \int_{y^2}^1 e^{\sqrt{x}} dx dy = ?$

(2) $\int \frac{x e^x}{(x+1)^2} dx = ?$

(3) $\int e^x \cos x dx = ?$

二、(10%) $f(x) = \exp\left(\frac{1}{2}x^2\right)$, find $f^{(n)}(0) = ?$ (n is a nature number)三、(10%) Find $\sum_{n=1}^{\infty} np^{n-1} = ?$ if $0 < p < 1$ 四、(10%) Find the extreme of $f(x, y, z) = x^2 + 2xy + yz^2$ subject to $2x + y + z^2 = 24$
and $x + z = 8$ 五、(10%) Find the interval of convergence for the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n} x^n$.

六、(15%) Let $A = \begin{bmatrix} 1 & 2 & 2 & -1 \\ 3 & 6 & 5 & 0 \\ 1 & 2 & 1 & 2 \end{bmatrix}$

(1) Find a basis for the null space of A .(2) Find a basis for the column space of A .(3) Find a basis for the row space of A .(4) Find the rank(A) and nullity(A).

七、(15%) Let $A = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$

(1) Find the eigenvalues and bases for the eigenspaces of A^{25} (2) Is A diagonalizable?八、(10%) Suppose that A is a 5×4 matrix and B is a 4×5 matrix. Prove that AB is not invertible.