19.

淡江大學 103 學年度碩士班招生考試試題

系別：數學學系　　　　科目：機率與統計

考試日期：3 月 2 日(星期日) 第 3 節　　　　本試題共　大題，　頁

1.(10%) Let the random variable X has the p.m.f. $f(x) = \frac{(|x|+1)^2}{9}$, $x = -1, 0, 1$ Compute $E(X), E(X^2)$, $E(3X^2 - 2X + 4)$ and $Var(X)$.

2. (15%) Let the joint p.m.f. of X and Y be

$$f(x, y) = 1/4, \quad (x, y) \in S = \{(0,0), (1,1), (1,-1), (2,0)\}.$$

i) Compute $E(X \mid Y = 0)$ and $Var(X \mid Y = 0)$.

ii) Find the covariance of X and Y.

iii) Prove or disprove that X and Y are independent.

3. (10%) Customers arrive in a certain shop according to an approximate Poisson process at mean rate of 20 persons/hour. Let X denote the waiting time in minutes until the first customer arrival. Find the p.d.f. of X and then compute $E(X)$ and $\mathrm{Var}(X)$.

4.(10%) Let $X_1, \cdots, X_n$ be iid r.v.'s from uniform $(0, \theta)$ where θ is an unknown parameter. Find a complete sufficient statistic for estimating θ and then find the UMVUE of θ.

5.(20 %) Let $Y_i = \alpha + \beta X_i + \epsilon_i$, $i = 1, \cdots, n$ be the equation between two random veriables X_i and Y_i. Let $\hat{\alpha}$, $\hat{\beta}$ be the least squares estimates of α and β, i.e. $\sum(Y_i - \hat{\alpha} - \hat{\beta}X_i)^2$ is a minimum.

a). Prove that $\hat{\beta} = S_{XY}/S_{XX}$, $\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$, where $S_{XY} = \sum[(Y_i - \bar{Y})(X_i - \bar{X})]$, $S_{XX} = \sum(X_i - \bar{X})^2$.

b). Show that $E\hat{\beta} = \beta$ and $E\hat{\alpha} = \alpha$.

6. (10%) Let $X_1, \cdots, X_n$ be iid from $B(1, p)$. For testing $H_0 : p \le 1/2$ v.s. $H_1 : p > 1/2$ at $\alpha = 0.05$. Apply the Central limit theorem to determine the sample size n so that the probability of rejecting H_0 when $p = 7/8$ is 0.95.

7. (25%) Let X_1, X_2 and X_3 be i.i.d. random variables from $N(\mu, 12)$. Consider the test for $H_0 : \mu = 0$ v.s. $H_1 : \mu = 1$ with significance level 0.05.

i). Construct a critical region A so that the test has level 0.05.

ii). Find a confidence lower bound $L(X_1, X_2, X_3)$ such that $P(L(X_1, X_2, X_3) \le \mu) = 0.95$.

iii). Are the two event "$\{X_1, X_2, X_3\} \in A$" and "$L(X_1, X_2, X_3) > 0$" equivalent ?

iv). Find the power of this test.

v). Compute the p-value if we observe $X_1 = 1, X_2 = 0.96$ and $X_3 = 1.96$