

國立高雄大學 103 學年度研究所碩士班招生考試試題

科目：離散數學與資料結構  
考試時間：100 分鐘

系所：資訊工程學系  
本科原始成績：100 分

是否使用計算機：否

1. The Stirling numbers of the second kind,  $S(n, k)$ , is the numbers of ways to distribute  $n$  distinct objects into  $k$  indistinguishable containers with every container receiving at least one object.

(a) (6%) Let  $m, n$  be positive integers with  $1 < n \leq m$ . Show that

$$S(m+1, n) = S(m, n-1) + nS(m, n).$$

(b) (4%) Calculate  $S(6, 3)$ .

2. (a) (3%) What is the Principle of Mathematical Induction?

(b) (3%) What is the Principle of Strong Mathematical Induction?

(c) (4%) For all  $n \in \mathbb{Z}^+$ , show that if  $n \geq 18$ , then  $n$  can be written as a sum of 4's and 7's, for example,  $18=7+7+4$ .

3. For 8-digit integers, (leading zeros allowed), how many of them have

(a) (4%) at most three distinct digits? (For example, use only, 0, 5, and 9)

(b) (3%) exactly three distinct digits?

(c) (3%) at least three distinct digits?

4. We can write a positive integer  $n$  as a sum of 1's and 2's, where the order of the summands is considered relevant, for example,  $4=1+1+2$ ,  $4=2+1+1$ , or  $4=2+2$ .

(a) (4%) Create a recurrence relation for counting the number of different ways to write  $n$  as a sum of 1's and 2's.

(b) (3%) Solve the recurrence relation in (a).

(c) (3%) In how many ways can 10 be written as a sum of 1's and 2's, where the order of the summands is considered relevant?

5. There are 6 faces of a die (骰子), labeled 1, 2, 3, 4, 5, and 6, respectively. We color the 6 faces such that two neighboring faces cannot have a same color.

(a) (3%) In how many ways, can this die be colored in 3 colors?

(b) (4%) In how many ways, can this die be colored in  $n$  colors,  $n \geq 3$ ?

(c) (3%) In how many ways, can this die be colored in 4 colors?

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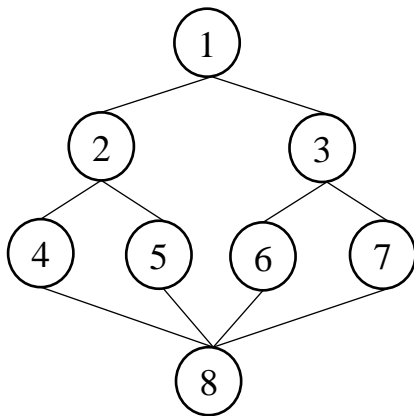
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6. Assume the following data type definition and declaration:

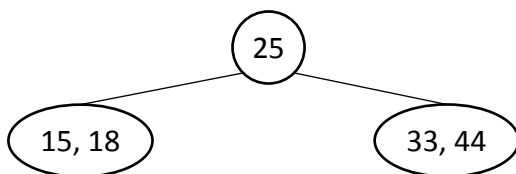
```
typedef struct node_s {  
    int num;  
    struct node_s *restp;  
} node_t;  
...  
node_t *headp, *cur_nodep;
```

Please write a **for** loop header that causes **cur\_nodep** to point in succession to each node of the linked list whose initial pointer is stored in **headp**. The loop should exit when **cur\_nodep** reaches the end of the list. (5%)

7. Given the following graph, please show step-by-step how to print the value of each vertex by stacks for the Depth-First Search method. (10%)



8. (a) Please show the following 2-3 tree after inserting 45 and then 20. (5%)



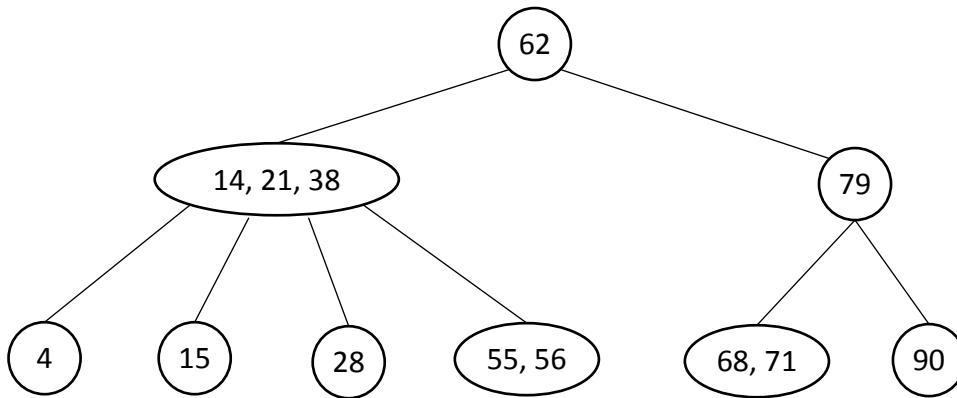
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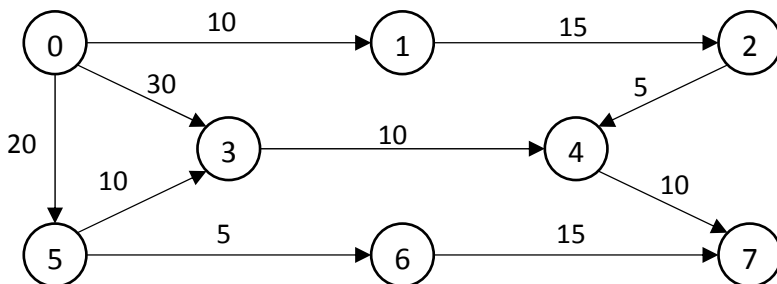
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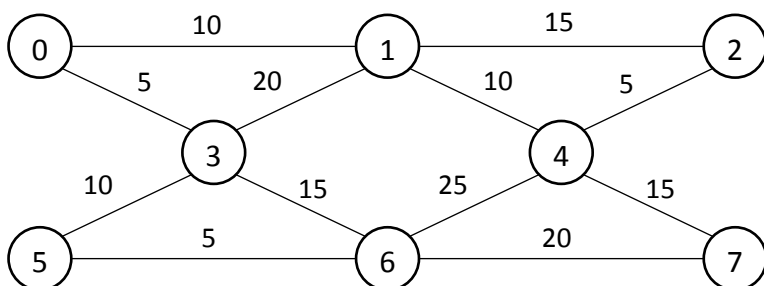
(b) Please show the following 2-3-4 tree after inserting 40 and then 50. (5%)



9. For the following graph, please find the shortest path from 0 to all other vertices. (10%)



10. For the following graph, please find the minimum spanning tree. (10%)



11. Given  $n$  nodes, please show how many different binary trees you can construct. (5%)