國立高雄大學 103 學年度研究所碩士班招生考試試題

科目：工程數學
考試時間：100 分鐘

系所：
土木與環境工程學系(土木工程組)
本科原始成績：100 分

是否使用計算機：是

Problem Set 1

Consider, as a simple structure, the column shown in Fig.1. The column remains straight until a certain value of P is reached, which we call the critical load or buckling load and which we denoted as P_{cr}. Euler beam theory tells us that the lateral deflection $y(x)$ is governed by the boundary value problem

$$EIy'' + Py = 0,\ 0<x<L \qquad (1a)$$

$$y(0) = 0, y(L) = 0 \qquad (1b)$$

where E is Young's modulus of the material and I is the cross section inertia.

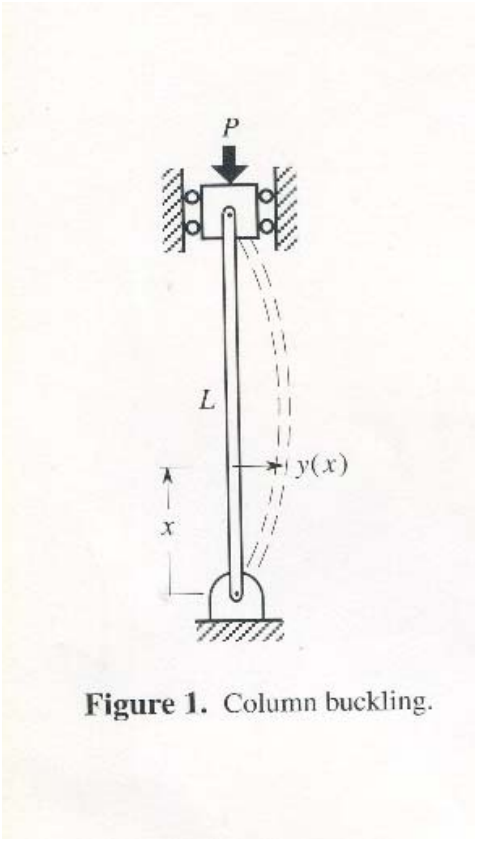

Figure 1. Column buckling.

(a) Show the nontrivial solution occurs for the eigenvalue $\lambda = P/EI = \pi^2/L^2, 4\pi^2/L^2, 9\pi^2/L^2, \ldots$(15%)

(b) Find the buckling load P_{cr} using the smallest eignevalue (10%)

Problem Set 2

Consider the ordinary differential equation shown below:

$$y(x)''-6y'(x)+9y=0 \qquad (2)$$

(a) Show $y = e^{3x}$ is one solution (5%).

(b) Use the change of variable $y(x) = e^{3x}u(x)$ to find the other solution (15%),

(c) Find the general solution (5%).

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Problem Set 3

The following system of linear equations has four unknowns, x_1, x_2, x_3, and x_4

$$3x_1 + 2x_2 + 2x_3 - 5x_4 = 8$$
$$2x_1 + 5x_2 + 5x_3 - 18x_4 = 9$$
$$4x_1 - x_2 - x_3 + 8x_4 = 7$$

(a) Determine the ranks of the matrix of the coefficient A, and the augmented matrix B.(10%)
(b) How many linear independent row vectors do A have?(5%)
(c) Find the relationship between x_1, x_2, x_3, and x_4 if there are any.(10%)

Problem Set 4

$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ and V_1, V_2, V_3, and V_4 are the four distinct eigenvectors of A.

$$V_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \quad V_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad V_4 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

(a) Determine the corresponding eigenvalues.(15%)
(b) Show the eigenvalue equation.(10%)