

國立高雄大學 103 學年度研究所碩士班招生考試試題

科目：工程數學  
考試時間：100 分鐘

系所：  
電機工程學系(光電領域)  
本科原始成績：100 分

是否使用計算機：是

請按次序作答

1. (10%)  $A = \begin{pmatrix} 2 & 8 \\ 0 & 4 \end{pmatrix}$ , find a diagonal matrix  $D$  that is similar to  $A$ .

2. (20%)  $B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{pmatrix}$ , (a) find the eigenvalues and the corresponded eigenvectors.

(b) Find the determinant of  $B$ . (c) Find the inverse of  $B$ .

3. (10%)  $C = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}$ , (a) find the rank of  $C$ . (b) Can the vectors  $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$

form a basis for  $R^3$ ? Please discuss

4. (10%)  $X' = \begin{pmatrix} 2 & 1 \\ 0 & 4 \end{pmatrix} X$ , solve  $X$

5. (10%)  $\frac{dy}{dx} = \frac{y}{x + \sqrt{xy}}$ , solve  $y(x)$

6. (10%)  $\frac{dy}{dx} = x + 5y$ , solve  $y(x)$

7. (10%)  $y'' - y' - 12y = e^{4x}$ , solve  $y(x)$

8. (10%)  $y'' + 9y = \begin{cases} \sin(3x), & 0 \leq x < \pi \\ 0, & x \geq \pi \end{cases}$ ,  $y(0) = 0, y'(0) = 1$ , solve  $y(x)$

9. (10%)  $y'' - (1+x)y' - y = 0$ , find the series solutions about  $x=0$ , write down the first three nonzero coefficients for each solution.