

國立高雄大學 103 學年度研究所碩士班招生考試試題

科目：微積分  
考試時間：100 分鐘

系所：應用數學系  
身份別：一般生、在職生  
本科原始成績：100 分

是否使用計算機：否

1 a. (8%) State the Fundamental Theorem of Calculus, part1 and part2.

b. (5%) Let  $g(x) = \int_{\cos x}^{5x} \cos(u^2)du$ . Find  $g'(x)$ .

2 (10%) Find the limit.

a.  $\lim_{x \rightarrow 0^+} x^{\sqrt{x}}$       b.  $\lim_{n \rightarrow \infty} \left( \frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{n}\sqrt{n+n}} \right)$

3 (10%) Find the derivative of the function.

a.  $f(x) = x^{\sin x}$       b.  $f(x) = \sin^{-1} \sqrt{\sin x}$

4 (10%) Evaluate the integral.

a.  $\int \frac{\cos(\ln x)}{x} dx$       b.  $\int_0^1 x5^x dx$

5 (10%) Show that if  $f$  is differentiable at  $a$  then  $f$  is continuous at  $a$ .

6 (10%) Let  $\mathcal{S}$  be the solid obtained by rotating  $\mathcal{R} = \{(x, y) | x \geq 1, 0 \leq y \leq 1/x\}$  about the x-axis.

a. Find the volume of  $\mathcal{S}$ .

b. Show that the surface area of  $\mathcal{S}$  is infinite.

7 (10%) For what values of  $x$  does the series  $\sum_{n=1}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$  converge?

8 (7%) Find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  if  $x^3 + y^3 + z^3 + 6xyz = 1$ .

9 (10%) Find the local maximum and minimum values and saddle points of

$$f(x, y) = x^4 + y^4 - 4xy + 1.$$

10 (10%) Evaluate  $\iint_D xy dA$ , where  $D$  is the region bounded by the line  $y = x - 1$  and the parabola  $y^2 = 2x + 6$ .