1. (10%) Show that $\mathbf{v}_1 = (2,12,8), \mathbf{v}_2 = (2,4,1), \mathbf{v}_3 = (-2,4,10)$ and $\mathbf{w}_1 = (1,-2,-5), \mathbf{w}_2 = (0,16,18)$ span the same subspace of $\mathcal{R}^3$, i.e. show that

$$span(\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}) = span(\{\mathbf{w}_1, \mathbf{w}_2\})$$

and that it is a subspace of $\mathcal{R}^3$.

2. (10%) Let $A$ and $B$ be invertible $m \times m$ matrices such that $A^{-1} + B^{-1}$ is also invertible, show that $A + B$ is also invertible. What is $(A + B)^{-1}$?

3. (10%) Use the *Laplace transform* to solve the integral equation

$$f(t) = \cos t + \int_0^t e^{-\tau} f(t-\tau)\, d\tau$$

4. (10%) Use separation of variables to find a solution of

$$x\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$$

5. (10%) Solve the initial-value problem

$$x^2y'' - 5x\,y' + 10y = 0, \quad y(1) = 1, y'(1) = 0$$

6. (10 %) For a 3x3 matrix

$$\begin{bmatrix} 2 & -1 & 0 \\ 5 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix},$$

Find the eigenvalues and eignevectors of this matrix.

7. (15 %) Use Green's Theorem to evaluate the line integral

$$I = \oint_C \frac{y^3}{3}\,dx + (xy + xy^2)dy$$

Where C is the count-clockwise boundary of the region in the first quadrant determined by the graphs of $y = 0$, $x = y^2$, $x = 1 - y^2$.

8. (15%) Expand $f(x) = x^2$, $0 < x < 1$ (a) in a Fourier series, (b) in a cosine series, (c) in a sine series, (d) plot the graph of the function in (a), (b), (c) within the region $-8 < x < 8$.

9. (10%) Evaluate the Cauchy principal value of

$$\int_{-\infty}^{\infty} \frac{\sin x}{x(x^2 - 2x + 2)}\,dx$$