

國立中山大學 103 學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【海下所碩士班】

題號：454001

※本科目依簡章規定「不可以」使用計算機

共 2 頁第 1 頁

總共 10 題，總分 100 分。

1. (10%)

Find a polynomial of the form  $y = a_0 + a_1x + a_2x^2$  which passes through the points  $(-1, 12)$ ,  $(0, 1)$ ,  $(1, 4)$ .

2. (10%)

For which rational numbers  $k$  does the homogeneous system

$$x + (k-3)y = 0$$

$$(k-3)x + y = 0$$

have a non-trivial solution?

3. (15%)

Let  $A, B, C, D$  be matrices defined by

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ -4 & 1 & 3 \end{bmatrix}, C = \begin{bmatrix} -3 & -1 \\ 2 & 1 \\ 4 & 3 \end{bmatrix}, D = \begin{bmatrix} 4 & -1 \\ 2 & 0 \end{bmatrix}$$

(a) Which of the following matrices are defined? (5%)

$$A+B, A+C, AB, BA, CD, DC, D^2$$

(b) Compute those matrices which are defined. (10%)

4. (5%)

Find the rational number  $k$  for which the matrix  $A$  is singular.

$$A = \begin{bmatrix} 1 & 2 & k \\ 3 & -1 & 1 \\ 5 & 3 & -5 \end{bmatrix}$$

5. (10%)

Let  $A = (2, 9, 8)$ ,  $B = (6, 4, -2)$ ,  $C = (7, 15, 7)$ .

(a) Show that the vectors  $AB$  and  $AC$  are perpendicular. (5%)

(b) Find the point  $D$  such that  $ABDC$  forms a rectangle. (5%)

6. (10%)

Solve the Ordinary Differential Equation

$$3x \frac{dy}{dx} + y + x^2y^4 = 0$$

背面有題

國立中山大學 103 學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【海下所碩士班】

題號：454001

※本科目依簡章規定「不可以」使用計算機

共 2 頁第 2 頁

7. (10%)

Solve the Ordinary Differential Equation

$$y''' - y'' + 2y = 0$$

8. (13%)

$$\text{If } f(x) = \begin{cases} 1 & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$$

(a) Find the Fourier cosine transform of  $f(x)$ . (8%)

(b) Hence from (a) evaluate the integral  $\int_0^{\infty} \frac{\sin 2ax}{x} dx$ . (5%)

9. (8%)

Find the Fourier series expansion of  $f(x) = x^2$ ,  $0 < x < 2\pi$ , with  $f(x + 2\pi) = f(x)$ .

10. (9%)

(a) Find the Fourier series expansion of  $f(x) = x$ ,  $-\pi < x < \pi$ . (5%)

(b) Use the results obtained in (a) to show that  $\frac{\pi^2}{12} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$ . (4%)