1. **(25%)** Classify the following differential equations, i.e. state their order, degree and linearity, and indicate the independent variable(s) and dependent variable(s) of the equations. DO NOT attempt to solve them.

   (a) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 10sinx$ **(5%)**

   (b) $\frac{dQ}{dt} = e^Q - 1$ **(5%)**

   (c) $cosx\left(\frac{dy}{dx}\right)^6 + sinx\left(\frac{d^2y}{dx^2}\right)^3 = 0$ **(5%)**

   (d) $(\frac{d^5x}{dt^5})^2 = x$ **(5%)**

   (e) $\frac{\partial^2u}{\partial x^2} + \frac{\partial^2u}{\partial y^2} = 0$ **(5%)**

2. **(15%)** A spring-mass-dashpot system. 2^{nd} order homogeneous linear ordinary differential equation (ODE) is defined as: $\mathbf{y'' + ay' + by = 0}$, where coefficients a *and* b are constant.
   (a) Find the characteristic equation of the system, **(4%)**
   (b) If the roots (r_1 and r_2) of the characteristic equation are real and unequal ($r_1 \neq r_2$), find the general solution, **(4%)**
   (c) If the coefficients a= 4 and b= 3, $y(0)$ =1, and $y'(0)$ = 0, find the particular solution, **(4 %)**
   (d) and draw a curve (y v.s. t, $0 < t < 2$) roughly representing the particular solution. **(3%)**
   (hint : try a solution of the function, $\mathbf{y = e^{rt}}$, where y: displacement, t: time, and r: root in the case of a spring-mass-dashpot system)

3. **(10%)** Laplace transform. (a) Define the Laplace transform (or called Laplace integral) $F(s)$ of the function $f(t)$, $t \geq 0$ **(5%)** and (b) let $f(t) = e^{at}$ when $t \geq 0$, where a is a constant, find $F(s)$. **(5%)**

4. **(50%)** This problem is regarding a heat transfer problem for a wire. A metal wire subjected to internal and external heating/cooling conditions so that the temperature of this wire is varied with respect to both time and space. Supposed this heat transfer problem can be described by the following PDE:

   $u_t = u_{xx} - \beta u - \alpha u_x + g(u,x)$

   with the initial and boundary conditions

   IC : $u(x,0) = f(x)$,

   BCs: $u(0,t) = a$; $u_x(\pi,t) = b$,

   where $u(x,t)$ is the time evolution of the temperature distribution.

   (a) What is a PDE ? For this problem, what are the possible physical meanings of the term u_{xx} and $u_x(\pi,t) = b$. **(10%)**

   (b) Why we call certain needed contition as 'boundary condition' ? **(5%)**

(c) If the parameters $\alpha = 0$, $\beta = 0$ and $g(u,x) = 0$ in the PDE as well as $a = b = 0$ and $f(x) = \sin(x)e^{-x}$ in the initial and boundary conditions, what is the temperature profile when time goes to infinity, why ? **(10%)**

(d) If one of the parameter $g(u,x)$ is $\sin(3x)$ and the other parameters remain the same, briefly describe how to solve this PDE. [You don't have to actually obtain the solution.] **(15%)**

(e) It is well known that Laplace transform (LT) is a very convenient and powful tool to transform a hard problem into an easier one, rendering it a popular method to solve the ordinary/partial differential equations. If one follows the paramters in (d) and $g(u,x) = u^2$, please write down your opinions whether LT is a suitable tool to tackle this problem. If yes, why and how to do attack this problem ? If no, what are the reasons ? **(10%)**