# 國立中正大學 101 學年度碩士班招生考試試題

系所別：機械工程學系-甲組、乙組、丙組　　科目：工程數學

第 1 節



1. (25%) Derive the general solution.

(a) $\frac{d^5y}{dx^5}-\frac{d^3y}{dx^3}=2x^2$ (15%)　　(b) $x^3\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}-4xy=1$ (10%)

2. (15%) Given $f(x)=\begin{cases}0 & for\ -3\le x\le 0\\ x & for\ 0\le x\le 3\end{cases}$. Find the Fourier series of $f(x)$ on the interval [-3, 3].

3. (10%) Find the Fourier integral of the given function $g(x)$ below.

$$g(x)=\begin{cases}\sin x & for\ |x|<\pi\\ 0 & for\ |x|>\pi\end{cases}$$

4. (15%) Let $B=\begin{bmatrix}11 & -8 & 4\\ -8 & -1 & -2\\ 4 & -2 & -4\end{bmatrix}$ and one eigenvalue of B is -5.

(a) Find all eigenvalues of $B$. (5%)

(b) Find a maximal set $S$ of nonzero orthogonal eigenvectors of $B$. (5%)

(c) Find an orthogonal matrix $P$ such that $D=P^{-1}BP$ is diagonal. (5%)

5. (10%) Find an orthogonal basis and then an orthnormal basis for the subspace $U$ of $R^3$ spanned by $v_1=(1,1,1)$, $v_2=(1,2,4)$, and $v_3=(1,-3,-4)$.

6. (25%) Solve the one dimensional wave equation with the boundary conditions and initial conditions as given below.

$$\frac{\partial^2u}{\partial t^2}=\frac{1}{\pi^2}\frac{\partial^2u}{\partial x^2},\ 0<x<1,\ t>0$$

$$u(0,t)=0,\ t>0;\qquad u(1,t)=0,\ t>0;$$

$$u(x,0)=\sin\pi x\cos\pi x,\ 0<x<1;$$

$$\frac{\partial u}{\partial t}(x,0)=0,\ 0<x<1;$$

using the Method of Separation of Variables.