1. (10%) Compute and plot the convolution of the following two signals:

$$x(t)=\begin{cases}1, & 0<t<3\\ 0, & \text{otherwise}\end{cases}$$

$$h(t)=\begin{cases}t, & 0<t<6\\ 0, & \text{otherwise}\end{cases}.$$

2. (10%) Consider an LTI system whose response to the signal $x_1(t)$ in Figure 1(a) is the signal $y_1(t)$ illustrated in Figure 1(b). Determine and sketch the response of the system to the input $x_2(t)$ shown in Figure 1(c).

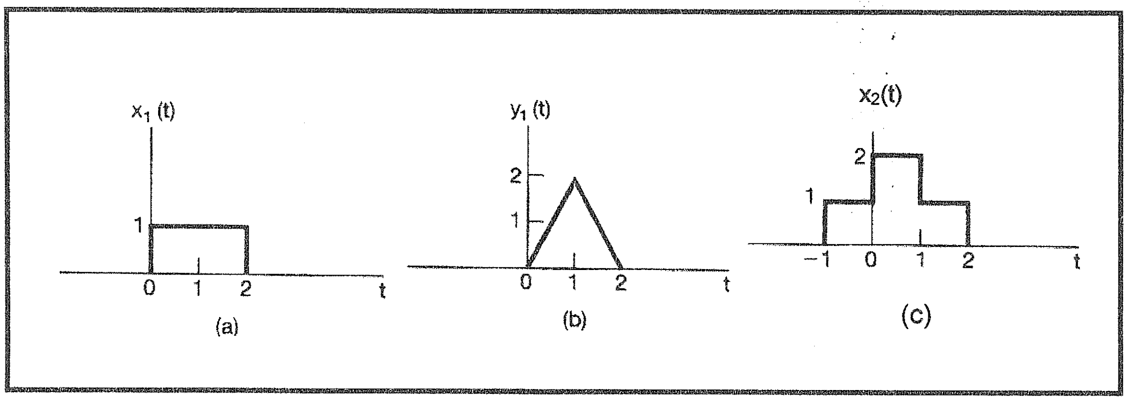

Figure 1.

3. (25%) Consider an LTI system whose response to the input

$$x(t)=[e^{-t}+e^{-3t}]u(t)$$

is

$$y(t)=[2e^{-t}-2e^{-4t}]u(t)$$

(A) (10%) Find the Fourier Transform of $x(t)$ and $y(t)$.

(B) (10%) Determine the frequency response and impulse response of this system.

(C) (5%) Find the differential equation relating the input and the output of this system.

4. (20%) Consider a causal LTI system, described by the difference equation

$$y[n]+\frac{1}{15}y[n-1]-\frac{2}{5}y[n-2]=x[n].$$

(A) (10%) Find the impulse response of the system.

(B) (5%) Determine the general form of the homogeneous solution to this equation.

(C) (5%) Find a particular solution to the difference equation when $x[n]=(3/5)^n u[n]$.

5. (25%) Consider the signal

$$x[n]=1+\sin\left(\frac{2\pi}{N}\right)n+3\cos\left(\frac{2\pi}{N}\right)n+\cos\left(\frac{4\pi}{N}n+\frac{\pi}{2}\right)$$

(A) (10%) Determine the Fourier series coefficients of $x[n]$.

(B) (8%) Plot the magnitude and phase of each set of Fourier series coefficients a_k.

(C) (7%) If $x[n]$ is the input to a causal discrete-time LTI system, described by the following difference equation:

$$y[n]-\frac{1}{4}y[n-1]=\frac{1}{2}x[n].$$

Find the Fourier series representation of the output $y[n]$.

6. (10%) Draw block diagram representations for causal LTI systems described by the following difference/differential equations:

(A) (5%) $y[n]=\frac{1}{3}y[n-1]+\frac{1}{5}x[n-1]$

(B) (5%) $\frac{dy(t)}{dt}+3y(t)=\frac{1}{2}x(t)$