Note : ε_0=8.854x10^{-12} F/m μ_0=1.26x10^{-6} H/m

1. (15%) (a) Please determine the expression for the capacitance C of a metallic sphere of radius a situated in air.

   (b) In terms of the resistance R between the metallic sphere and the grounding reference in air (at infinity), the corresponding relaxation time is $RC = \frac{\varepsilon_0}{\sigma}$. If the metallic sphere of radius 6 cm is buried in a homogeneous earth of conductivity $\sigma = 10^{-3} S/m$ and relative dielectric constant $\varepsilon_0 = 3$, its center is at a depth 5m with respect to the ground surface. Please determine the resistance R between the metallic sphere and the grounding reference in earth.

   (c) A 12 kV potential with respect to the reference point at infinity supports a steady current flowing through the metallic sphere of (b), please determine the tangential component of the electric field intensity vector on the earth surface using the image theory and superposition principle.

2. (15%) As seen in Fig.1, a conducting cone (angle θ_0) is placed on a grounded conducting plane with a tiny gap separating it from the plane. The cone is maintained at a voltage V_0 Volts.

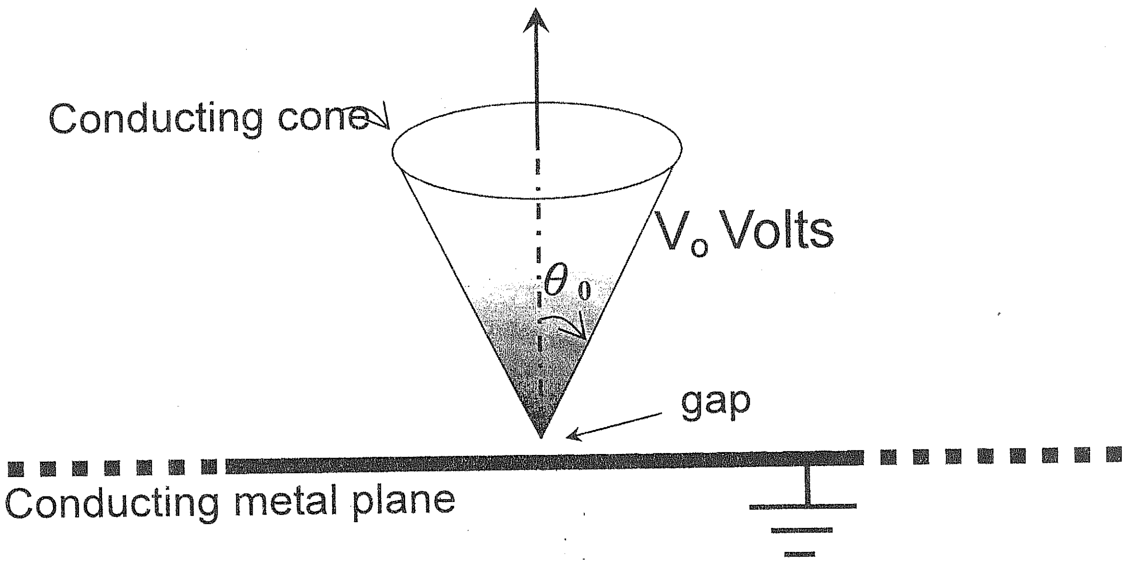

Fig. 1

(5%)(a) Use Laplace's equation to find the electric potential V (r, θ, φ) in the region $\theta_0 < \theta < \pi$

(ref : $\int \frac{d\theta}{\sin\theta} = \ln(\tan\frac{\theta}{2}) + C$)

(10%)(b) Please also derive the electric filed in the region $\theta_0 < \theta < \pi$ and the charge density on the two conductors

3. (10%) The polarization vector of a dielectric rod with radius 10cm along the y-axis from y=0 to 5m is $\bar{P}=(y^2+5)\hat{y}$,
   (a) Please derive the bound volume charge density and the surface polarization charge on each surface,
   (b) How about the total bound charge?

4. (10%) A wire bent as shown in Fig. 2, lies in the *xy*-plane and carries a current *I* flowing from $-(a+L)$ to $(a+L)$. If the magnetic flux density in the region is $B\hat{z}$, please to determine the magnetic force acting on the wire.

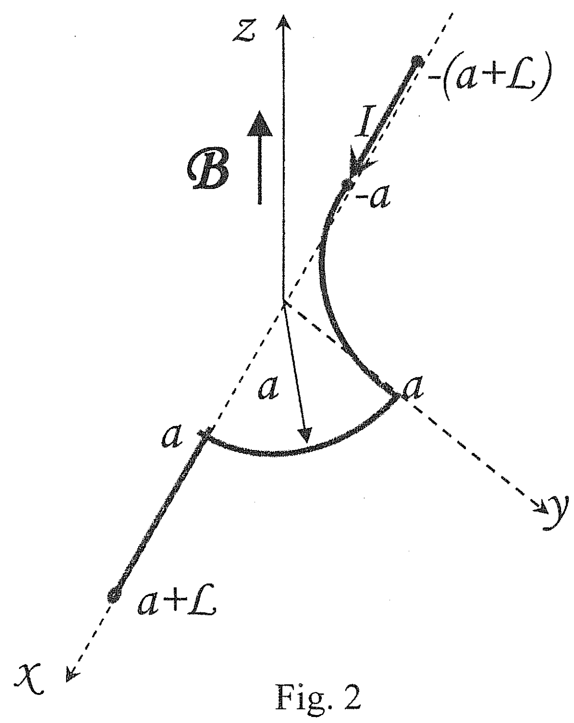

Fig. 2

5. (15%) Consider a standard air-filled rectangular waveguide with dimensions ***a***=8 cm, ***b***=4 cm and ***L***=6 m, as shown in Fig. 3.
   (a) What wave modes can propagate along this waveguide at a frequency of f=4.5 GHz ?
   (b) What is the dominant (operating) frequency range of this waveguide?
   (c) If two signals (f_1=2 GHz, f_2=3 GHz) are launched at the same instant of time at one end of the waveguide, find the time lag between the two signals as they are received on the other end of the waveguide.

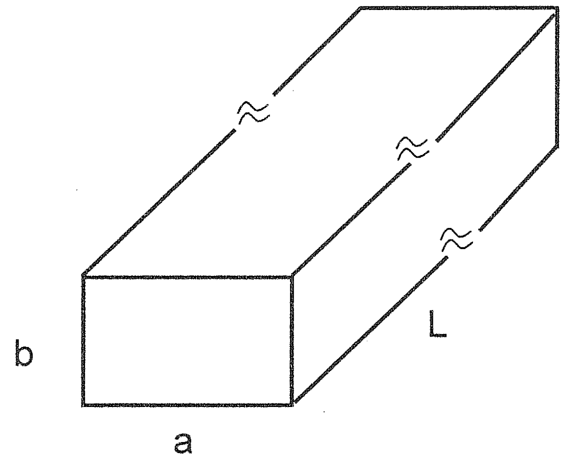

Fig. 3

6. (15%) The instantaneous magnetic field intensity vector of a wave propagating in a lossy non-magnetic medium (ε_r=10, μ_r=1) is given by

$$\vec{H}(t,x) = 5e^{-\alpha x}[\cos(2.4\times10^{8}t - 2.83x)\hat{z} + \sin(2.4\times10^{8}t - 2.83x)\hat{y}]$$

(a) Determine the attenuation coefficient α ?

(b) Determine the instantaneous electric field intensity vector ?

(c) If this wave is incident normally on a perfect conductor plane, please determine the polarization of the reflected wave?

7. (8%) A radar antenna radiates a 20-GHz electromagnetic wave with the amplitude of the electric field intensity approximately given by the following function of the distance ***r*** from the radar:

$$E_m(r) = (5/r) \quad (kV/m)$$

According to IEEE standard, the safety limits for human exposure to electromagnetic radiation allows the maximum time-average intensity of the Poynting vector in uncontrolled environment is $(P_{avg})_{\max} = 100\ W/m$ for frequencies between 15 GHz and 300 GHz.

(a)Based on this standard, what is the radius of the unsafe zone around the antenna?

(b) From your opinion, for the frequencies below 15 GHz, the maximum intensity of the safety limit should be increased or reduced? Briefly explain your reason.

8. (12%) Write the differential form of time-harmonic Maxwell's equations