1. (8%) Solve the following linear system, where a, b, and c are constants:

$$\begin{aligned} x_1 + x_2 + x_3 &= a \\ 2x_1 + 2x_3 &= b \\ 3x_2 + 3x_3 &= c. \end{aligned}$$

2. (7%) Prove that (x_1, y_1), (x_2, y_2), and (x_3, y_3) are collinear points if and only if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$$

3. (10%) Determine the standard matrix for the orthogonal projection of the vector $\mathbf{x} = (x, y)$ on the line L that makes an angle θ with the positive x-axis in R^2.

4. Let $T : P_2 \rightarrow P_2$ be the linear operator defined by $T(p(x)) = p(2x + 5)$, that is, $T(c_0 + c_1x + c_2x^2) = c_0 + c_1(2x + 5) + c_2(2x + 5)^2$.

   (a) (5%) Find the matrix for T with respect to the basis $B = \{1, x, x^2\}$

   (b) (5%) Is T one-to-one? If so, find the matrix for T^{-1} with respect to the basis B.

5. Given the matrix A as

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

   (a) (5%) Is A positive definite, negative definite, or indefinite? Why?

   (b) (5%) If A is the standard matrix for a linear transformation T, what is the kernel of T? What is the range of T?

6. (5%) Find the distance between the point $P(2, 1, -1)$ and the plane W, where W is defined by the equation $3x - 4y + z = 0$.

7. Let S be a non-empty finite set of symbols, and $P(S)$ be the set of all permutations of symbols in S.
   (a) (5%) Give a recursive definition of $P(S)$.
   (b) (5%) Proof by induction that $|P(S)| = n!$, where $n = |S|$.
   (c) (5 %) Give a recursive procedure to compute $P(S)$.

8. Give the answer to each of the following problems.
   (a) (3 %) Count the number of bit strings of length 10 that have either 011 as a prefix (字串起始) or 01 as a suffix (字串結尾).
   (b) (3 %) Translate the following English statement into a logical expression in proposition logic: You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.
   (c) (4 %) Use extended Euclidean algorithm to find an inverse of 3 modulo 7.

9. A drawer contains 7 brown socks, 9 white socks, and 11 black socks, all unmatched. A man takes socks out at random in the dark.
   (a) (5%) How many socks must he take out to be sure that he has at least two socks of the same color?
   (b) (5%) How many socks must he take out to be sure that he has at least two brown socks?

10. A simple graph is called regular if every vertex of this graph has the same degree. A regular graph G of degree $\boldsymbol{m}$ has $\boldsymbol{n}$ vertices.
   (a) (5%) How many edges does G have?
   (b) (5%) The complementary graph $\overline{G}$ of G has the same vertices as G. Two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. How many edges does $\overline{G}$ have?
   (c) (5%) An Euler circuit in G is a simple circuit containing every edge of G. For which values of $\boldsymbol{m}$ *and/or* $\boldsymbol{n}$ does G have an Euler circuit?