系所組別：土木工程學系丙、戊組
考試科目：工程統計
考試日期：0222，節次：3

※ 考生請注意：本試題可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Suppose X is a continuous random variable with a probability density function as follows. Assume that E(X) = 3/5
(1) Find the value of a and b. (2) Find the variance of X. (5 points each)

$$f(x) = \begin{cases} a + bx^2 & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

2. The compressive strength of samples of cement can be modeled by a normal distribution with a mean of 3000 kg/cm^2 and a standard deviation of 50 kg/cm^2. (2 points each)
   (1) What is the probability that a sample's strength is less than 3150 kg/cm^2?
   (2) What is the probability that a sample's strength is between 2800 and 2900 kg/cm^2?
   (3) What strength is exceeded by 95% of the samples?
   (4) What is the median strength?
   (5) What is the probability that strength exceeds the mean strength by more than 1 standard deviations?

Values Provided for Your Calculations

z	-1.00	0.8	1	1.64	1.96	2	2.17	3	$t_{0.025, 10}$	$t_{0.025, 24}$	$t_{0.05, 25}$	$t_{0.025, 25}$
Φ(z)	0.159	0.788	0.841	0.95	0.975	0.977	0.985	0.998	2.228	2.064	1.708	3.060

3. Suppose that the actual amount of instant coffee a filling machine puts into "6-ounce" cans varies from can to can and that the actual fill may be considered a random variable having a normal distribution, with a standard deviation of 0.04 ounces. If on average only 3 out of every 200 cans contain less than 6 ounces of coffee, what must be the mean fill of these cans?: (5 points)

4. Consider a random sample $X_1, \ldots, X_n$ from the probability density function as follows: $f(x;\ \theta) = 0.5\bullet(1+\theta\bullet x)$ if $-1 \le x \le 1$ where $-1 \le \theta \le 1$. Show that $\hat{\theta} = 3\overline{x}$ is an unbiased estimator of θ. (2 points)

5. Pairs of *P*-values and significance levels, α, are given. For each pair, state whether the observed *P*-value would lead to rejection of H_0 at the given significance level. (1) *P*-value = 0.084, α = 0.05, (2) *P*-value = 0.003, α = 0.01, (3) *P*-value = 0.498, α = 0.05, (4) *P*-value = 0.084, α = 0.10, (5) *P*-value = 0.039, α = 0.01, (6) *P*-value = 0.218, α = 0.10, (5 points each)

6. Let be the mean of a random sample of size *n* from a distribution N(μ, 9). Find *n* such that P(−1 < μ < +1) = 0.95. (5 points)

7. In a test of $H_0: \mu = 100$ vs. $H_a: \mu \ne 100$, a sample of size 80 produces z = 0.8 for the value of the test statistic. Calculate the *P*-value of the test. (2 points)

8. The government wants to estimate the mean weekly family expenditure for food (μ). The government assumes that weekly family expenditure for food is normally distributed but does not know the standard deviation. The government takes a simple random sample of 25 families and finds that the sample mean, $\bar{x}$, is $100 and the standard deviation of the sample, s, is $31. (1) Find a 95% confidence interval for the mean weekly family expenditure for food. (2) The government wants to test the hypothesis $H_0: \mu = 95$ against $H_a: \mu \neq 95$. Carry out the test at the 0.05 significance level. Report whether or not the null hypothesis is rejected or not. (5 points each)

9. Data were obtained in a study of sales volume per district (Y) as a function of the number of client contacts per month (X). The following quantities were computed from the data: n=12, $\Sigma x = 422$, $\Sigma x^2 = 18706$, $\Sigma y = 863$, $\Sigma y^2 = 86469$, $\Sigma xy = 39965$ (2 points each)

(1) Calculate S_{xx}, S_{yy}, and S_{xy}.

(2) Fit a linear regression $y = \beta_0 + \beta_1 \cdot x$ using the above data

(3) Compute the ANOVA table for the regression.

(4) Provide an estimator of σ^2.

(5) Compute a 95% confidence interval for β_1.

(6) Test the following hypothesis $H_0: \beta_1 = 0$ vs. $H_a: \beta_1 \neq 0$ at $\alpha = 0.05$ by examining the confidence interval constructed above.

(7) Calculate the coefficient of determination and give an interpretation in the context of the problem.

(8) Below in Figure (1) are the residual plots and the histogram for the estimated residuals from the regression. Answer the following questions based on these plots. Is the assumption that the random errors have constant variance violated? Why, why not?

(9) Is the assumption that the random errors have a normal distribution violated? Why, why not?

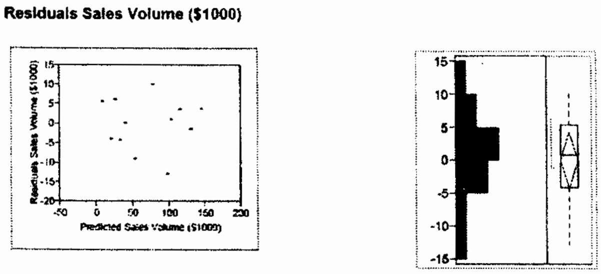

Figure (1) for Question 9

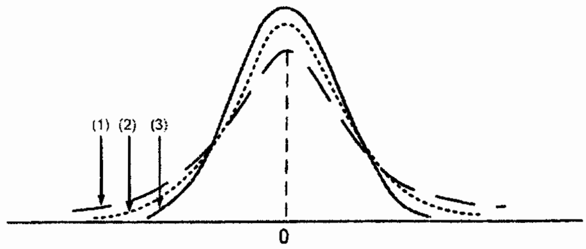

Figure (2) for Question 11

10. Select the best answer from the following choices and explain your rationale. We are interested in testing $H_0: \mu = 4$ vs. $H_a: \mu \neq 4$. Based on the two confidence intervals below, where does the *P*-value lie?

(1) 90% C.I.: (0.63, 3.6) (2) 95% C.I.: (0.03, 4.2)

(a) *P*-value < 0.01, (b) *P*-value < 0.05 (c) 0.05 < *P*-value < 0.1, (d) 0.01 < *P*-value < 0.05, (e) *P*-value > 0.05. (5 points)

11. There are three curves shown in Figure (2) for z-, t_5- and t_{25}-distributions, respectively. Identify these distribution curves corresponding to the numbers listed in the figure, and explain your rationale. (3 points)