

系所組別：土木工程學系甲、乙、丁組

考試科目：工程數學

考試日期：0222，節次：3

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve the following ordinary differential equations.

(a) $yy' + xy^2 = x^3$. (10%)

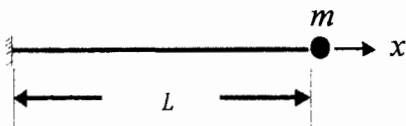
(b) $y'' - y = \frac{2}{e^x - 1}$ (10%)

2. Find the general solution of the following partial differential equations.

(a) $y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = 0$ (10%)

(b) $\frac{\partial^2 u}{\partial x^2} - 4 \frac{\partial^2 u}{\partial x \partial y} + 4 \frac{\partial^2 u}{\partial y^2} = 0$ (10%)

3. Problem: A bar has length L , density ρ , cross section area A , Young's modulus E . Its end $x = 0$ is fixed and a mass m is attached to its free end $x = L$. The bar initially is stretched linearly by moving m a distance δ to the right; at time $t = 0$ the system is released from rest. (Assume that ρ, A, E are constants.)

(a) Derive the governing equation of the axial displacement $u(x, t)$ of the bar. (10%)

(b) State the boundary conditions of this problem. (5%)

(c) State the initial conditions of this problem. (5%)

4. (a) Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic in some domain. (5%)(b) Find a harmonic conjugate $v(x, y)$. (5%)(c) For analytic function $f(z) = u(x, y) + iv(x, y)$, determine $f'(z)$. (10%)5. (a) Show that $\mathbf{F} = 2xy\mathbf{i} + (x^2 + 2yz)\mathbf{j} + (y^2 + 1)\mathbf{k}$ is a conservative vector field. (5%)(b) Find a scalar function $\phi(x, y, z)$, such that $\mathbf{F} = -\nabla\phi$. (5%)(c) Determine the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $C: \cos t\mathbf{i} + \sin t\mathbf{j} + 2t\mathbf{k}$, $0 \leq t \leq \frac{\pi}{2}$ (10%)