系所組別：機械工程學系甲乙丙丁戊組

考試科目：工程數學 考試日期：0222，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (i) What is the general solution x(t) of the differential equation $d^2x / dt^2 + 9x = 0$.(3%)
   (ii) Please give a boundary condition such that this boundary value problem, the differential equation in (i) plus this boundary condition, has unique solution.(3%)
   (iii) Please give a boundary condition such that this boundary value problem, the differential equation in (i) plus this boundary condition, has many solutions.(3%)
   (iv) Please give a boundary condition such that this boundary value problem, the differential equation in (i) plus this boundary condition, has no solution.(3%)

2. Evaluate the integral $\int\int_R x\,y\,dA$ over the region R, which is bounded by 4 curves $y = 4x^2$, $y = x^2$, $xy = 1$ and $xy = 5$ in the x, y domain. Please follow the guide line to answer this problem.
   (i) I would suggest using the change of variables $u = y/x^2$, $v = xy$. Plot the region R in the x, y domain and u, v domain.(4%)
   (ii) What is the Jacobian matrix of the mapping from x, y domain to the u, v domain.(4%)
   (iii) Evaluate the integral in the u, v domain to get the answer.(5%)

3. What is the definition of the conservative vector field?(5%) Determine whether the vector field $\mathbf{F}(x, y) = (y^2 - 6xy + 6)\mathbf{i} + (2xy - 3x^2 - 2y)\mathbf{j}$ is conservative.(5%)

4. Evaluate $\oint_c \frac{1}{z^2+1} dz$, c: (a) $|z+i| = 1$, (b) $|z-i| = 1$, counterclockwise. (15%)

5. Determine the eigenvalues, eigenvectors, generalized modal matrix and Jordan form of $A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$. (15%)

(背面仍有題目,請繼續作答)

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6. Briefly answer the following:

(i) For the ordinary differential equation, $x(x-1)y'' + 3y' - 2y = 0$, identify all singular points (if any) and classify each as regular singular or irregular singular. If there exist solutions of the form of power series about $x = 0$, determine the appropriate form of the general solutions. (Although you need NOT work out the series solutions, you are required to write down the correct form of the series, including the exponents (the indicial roots) of the singularity if any.) Determine the minimum radii of convergence of the series. (8%)

(ii) Consider a set of infinite number of real-valued functions $\{\phi_1(x), \phi_2(x), \phi_3(x), \cdots\}$ orthogonal with respect to a weighting function $p(x)$ on $[a, b]$. What do we mean that it is an orthogonal set? What do we mean that if it is a complete set? What is the importance (consequence) of the completeness? (7%)

(iii) Write out all the eigenvalues, λ, and the corresponding eigenfunctions of $(1-x^2)y'' - 2xy' + \lambda y = 0$, $y(0) = 0$, $y(1)$ bounded. (5%)

7. Consider a slightly damped vibrating string that satisfies

$$\rho_0 \frac{\partial^2 u}{\partial t^2} = T_0 \frac{\partial^2 u}{\partial x^2} - \beta \frac{\partial u}{\partial t}$$

Determine the solution (by separation of variables) which satisfies the boundary conditions

$$u(0,t) = 0 \quad \text{and} \quad u(L,t) = 0$$

and the initial conditions

$$u(x,0) = f(x) \quad \text{and} \quad \frac{\partial u}{\partial t}(x,0) = g(x)$$

Assume that the frictional coefficient β is relatively small ($\beta^2 < 4\pi^2 \rho_0 T_0 / L^2$). (15%)