系所組別：環境工程學系丙組

考試科目：微積分　　考試日期：0222，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Given $\lim_{x\to 0}\frac{\sin x}{x}=1$. Please determine the limit for the following functions. (15%)

   (1) $\lim_{x\to 0^+}\frac{\sin 4x}{5x}$　(2) $\lim_{x\to\infty} x\sin\frac{a}{x^2}$　(3) $\lim_{x\to 0}\frac{\sin x^2}{x}$

2. Please find the derivative of the following functions. (20%)

   (1) $y=x\arcsin x+\sqrt{1-x^2}$　(2) $y=\frac{(x+1)(x-2)}{(x-1)(x+2)}, x>2$　(3) $y=\ln\sqrt{2+\cos^2 x}$

   (4) $y=\operatorname{sech}^{-1}(\cos 2x), 0<x<\pi/4$

3. The normal probability density function (whose mean is μ and standard deviation is σ) is given by

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right].$$

   (1) Please find the extrema and the points of inflection of the normal probability density function. (5%)

   (2) Please evaluate the value of $\int_{-\infty}^{\infty} f(x)dx$ when μ=0 and σ=1. (Hint: use polar coordinates to evaluate the improper integral $I^2=\left(\int_{-\infty}^{\infty}e^{-x^2/2}dx\right)\left(\int_{-\infty}^{\infty}e^{-y^2/2}dy\right)$). (10%)

4. Please evaluate the integral of the following functions. (30%)

   (1) $\int\frac{1}{2x\sqrt{1-4x^2}}dx$　(2) $\int(\cos 3\theta-1)d\theta$　(3) $\int\frac{2x-5}{x^2+2x+2}dx$

   (4) $\int_0^{\infty}\frac{e^{-x}-e^{-2x}}{x}dx$ (*Hint*: Evaluate $\int_1^2 e^{-xy}dy$.)　(5) $\int_0^2\int_{2x-4}^{0}\frac{2y-1}{x+1}dydx$　(6) $\int_1^4\int_0^1\int_0^x 2ze^{-x^2}dydxdz$

5. Please sketch the region R and evaluate the iterated integral $\iint_R f(x,y)dA$ for the function given by:

$$\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}(x+y)dydx.$$ (10%)

6. Please find the volume of the solid inside the hemisphere $z=\sqrt{16-x^2-y^2}$ and outside the cylinder $x^2+y^2=1$. (10%)