

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please solve the following differential equations: (5 points for each one)

A.  $y'' - 5y' + 4y = \cos^2 x$                       B.  $y''' + y'' = e^x \cos x$

C.  $y'' + 4y' + 5y = \delta(t - 2\pi)$  with  $y(0) = 0, y'(0) = 0$

2. For two concentric spheres of radius  $r=a$  and  $r=b$ ,  $a < b$ , the temperature  $u(r)$  in the region between the spheres is

determined from the boundary-value problem  $r \frac{d^2 u}{dr^2} + 2 \frac{du}{dr} = 0$ ,  $u(a) = u_0$ ,  $u(b) = u_1$ , where  $u_0$  and  $u_1$  are constants. Please solve for  $u(r)$ . (10 points)

3. The input flow rate and concentration into a CSTR are  $Q$  and  $C_0$ , respectively and those of output are  $Q$  and  $C$ . If the volume of the CSTR is  $V$ , please compute the concentration as a function of time if the initial concentration in the CSTR is zero and the chemical reaction is of second order decay. (10 points)

4. For the first-order differential equation  $\frac{dy}{dx} = f(x, y)$ , please derive the truncation errors for the fourth order

Runge-Kutta method used in a single step and multiple steps. (15 points)

5. For the Laplace's equation  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ , please find the solutions for the following conditions. (15 points for each one)

A. 
$$\begin{cases} u(0, y) = 1, \lim_{x \rightarrow \infty} u(x, y) = 0, & 0 < y < 1 \\ \frac{\partial u}{\partial y} \Big|_{y=0} = 0, \frac{\partial u}{\partial y} \Big|_{y=1} = -u(x, 1), & x > 0 \end{cases}$$

B. 
$$\begin{cases} u(0, y) = 0, u(1, y) = 0, & 0 < y < 1 \\ \frac{\partial u}{\partial y} \Big|_{y=0} = u(x, 0), u(x, 1) = 1, & 0 < x < 1 \end{cases}$$

6. The Dufort-Frankel method for the partial differential equation  $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$  is

$$\frac{T_i^{n+1} - T_i^{n-1}}{2\Delta t} = \frac{T_{i+1}^n - (T_i^{n+1} + T_i^{n-1}) + T_{i-1}^n}{\Delta x^2},$$

(A) please derive the truncation error; (B) please derive the conditions for consistency. (20 points)