

系所組別：光電科學與工程學系甲、乙組

考試科目：工程數學

考試日期：0223，節次：3

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一、問答題 (不須計算過程，請於答案卷上按順序標示題號並寫入答案)

1. (20%) A skydiver and his equipment together weigh 192 pounds. Before the parachute is opened, there is an air drag equal to six times his velocity. Four seconds ( $t = 4$  s) after stepping from the plane, the skydiver opens the parachute, producing an air drag equal to three times the square of the velocity. The free-fall acceleration is  $32 \text{ ft/s}^2$ . Please answer the following questions:

- (a) What is the velocity of the skydiver,  $v(t)$ , for  $t \leq 4$  s? (4%)  
 (b) What is the distance that the skydiver has fallen,  $s(t)$ , for  $t \leq 4$  s? (3%)  
 (c) What is the velocity of the skydiver,  $v(t)$ , for  $t \geq 4$  s. (8%)  
 (d) What is the distance that the skydiver has fallen,  $s(t)$ , for  $t \geq 4$  s? (5%)

2. (4%)

If  $a^2 + b^2 + c^2 = 16$ ,  $x^2 + y^2 + z^2 = 6$ , and  $a, b, c, x, y, z$  are all real number,

what is the maximum for  $\begin{vmatrix} 2 & 1 & 1 \\ a & b & c \\ x & y & z \end{vmatrix}$  ?

3. (4%)

What is the curvature of a circle with radius of  $a$ ? (express the answer as a function of  $a$ )

4. (4%)

Find the unit normal to the surface  $2x^2 + 4yz - 5z^2 = -10$  at the point of  $P(3, -1, 2)$  =?

5. (4%)

Find the volume of the region bounded by the parabolic cylinders  $z = 4 - x^2$  and the planes:  $x = 0, y = 0, y = 6, z = 0$

6. (4%)

$f = \frac{\ln e^{2x+2y}}{r^3}$ . Where  $r$  is the distance between point  $(x, y, z)$  and the origin  $(0, 0, 0)$ . Find  $\nabla \times \nabla f = ?$

7. (4%)

Find the parabola  $y = ax^2 + bx + c$  that is closet (least square error) to the data points of  $(-2, 0), (-1, 0), (0, 1), (1, 0), (2, 0)$ .

(背面仍有題目，請繼續作答)

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8. (4%)

Find the function  $f = ?$  such that  $\nabla f = (3x^2 + y^3 + ye^{xy})\vec{i} + (-2y^2 + 3xy^2 + xe^{xy})\vec{j}$ .

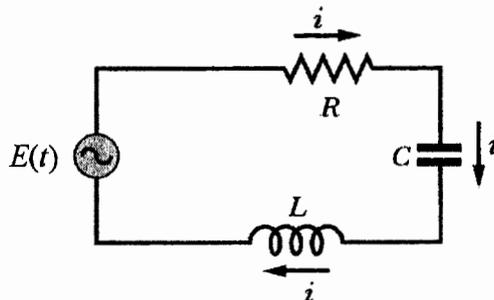
二、計算題 (須計算過程，請於答案卷上按順序標示題號並作答)

1. (15%) An object with a 16-pound weight is suspended from a spring, stretching it 8/11 feet. Then the object is submerged in a fluid that imposes a drag of  $2v$  pounds and  $v$  is the velocity of the object. The entire system is subjected to an external forces  $F(t) = 4\cos(\omega t)$ . Let  $y(t)$  be the displacement of the object from the equilibrium position ( $y = 0$ ) at time  $t$ .

(a) Find the displacement of the object in function of time, that is,  $y(t)$ . (10%)

(b) Determine the value of  $\omega$  that maximize the amplitude of the steady-state oscillation. (5%)

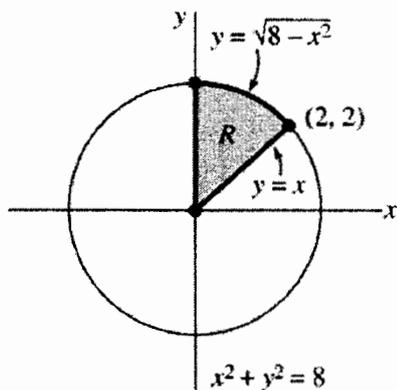
2. (15%) Determine the Fourier series representation of the steady-state current  $i(t)$  in the following  $RLC$  circuit (resistance  $R = 500 \Omega$ , inductance  $L = 5 \text{ H}$ , capacitance  $C = 0.2 \mu\text{F}$ ) if the applied electromotive force is  $E(t) = |10\sin(800\pi t)|$ .



3. (12%)

Use the figure below to evaluate the integral :

$$\int_0^2 \int_x^{\sqrt{8-x^2}} \frac{1}{5+x^2+y^2} dy dx$$



4. (10%)

Find the (a) eigenvalues and (b) eigenvectors of matrix  $A = \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$