

系所組別：電機資訊學院-資訊聯招

考試科目：機率統計

考試日期：0222，節次：3

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. A delivery truck travels from point A to point B and back using the same route each day. There are four traffic lights on the route. Let  $X_1$  denote the number of red lights the truck encounters going from A to B and  $X_2$  denote the number encountered on the return trip. Data collected over a long period suggest that the joint probability distribution for  $(X_1, X_2)$  is given by:

		$X_2$				
		0	1	2	3	4
$X_1$	0	0.01	0.01	0.03	0.07	0.01
	1	0.03	0.05	0.08	0.03	0.02
	2	0.03	0.11	0.15	0.01	0.01
	3	0.02	0.07	0.10	0.03	0.01
	4	0.01	0.06	0.03	0.01	0.01

- (1) Give the conditional density distribution of  $X_1$  given  $X_2 = 3$ . (5%)  
 (2) Give  $E(X_1)$ . (5%)  
 (3) Give  $E(X_1|X_2 = 3)$ . (5%)  
 (4) Give the standard deviation of  $X$ . (5%)
2. A chemical system that results from a chemical reaction has two important components among others in a blend. The joint distribution describing the proportion  $X_1$  and  $X_2$  of these two components is given by

$$f(x_1, x_2) = \begin{cases} 2, & 0 < x_1 < x_2 < 1 \\ 0, & \text{elsewhrer} \end{cases}$$

- (1) What is the probability that component proportions produce the results  $X_1 < 0.2$  and  $X_2 > 0.5$  (10%)  
 (2) Give the conditional distribution  $f_{x_1|x_2}(x_1|x_2)$  (10%)
3. The density function of the time  $Z$  in minutes between calls to an electrical supply store is given by

$$f(z) = \begin{cases} \frac{1}{10} e^{-z/10}, & 0 < z < \infty \\ 0, & \text{otherwise} \end{cases}$$

- (1) What is the mean time between calls? (5%)  
 (2) What is the variance in the time between calls? (7%)  
 (3) What is the probability that the time between calls exceeds the mean? (8%)

(背面仍有題目，請繼續作答)

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4. Suppose a filling machine is used to fill cartons with a liquid product. The specification that is strictly enforced for the filling machine is  $9 \pm 1.5$  oz. If any carton is produced with weight outside these bounds, it is considered by the supplier to be defective. It is hoped that at least 99% of cartons will meet these specifications. With the conditions  $\mu = 9$  and  $\sigma = 1$ ,
- (1) What proportion of cartons from the process are defective (See table A.3)? (10%)
  - (2) If changes are made to reduce variability, what must  $\sigma$  be reduced to in order to meet specifications with probability 0.99? Assume a normal distribution for the weight. (10%)
5. The random variables  $X$  and  $Y$ , representing the weights of creams and toffees, respectively, in 1-kilogram boxes of chocolates containing a mixture of creams, toffees, and cordials, have the joint density function

$$f(x,y) = \begin{cases} 24xy, & 0 \leq x \leq 1, 0 \leq y \leq 1, x+y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

- (1) Find the probability density function of the random variable  $Z=X+Y$ . (10%)
- (2) Using the density function of  $Z$ , find the probability that in a given box, the sum of the weight of creams and toffees accounts for at least  $1/2$  but less than  $3/4$  of the total weight. (10%)

