

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Please write down all your works on the answer sheet.

1. A function is defined by

$$f(x) = \int_0^{\pi} \cos t \cos(x-t) dt, \quad 0 \leq x \leq 2\pi$$

Find the minimum value of f . (10%)

2. Find an equation of the tangent line to the graph of the equation

$$4e^{xy} + \ln(x^2y + 1) = 4 \quad \text{at the point } (x, y) = (e, 0). \quad (10\%)$$

3. If n is a positive integer, find the following integrals. (20%)

$$(a) \int_0^1 (\ln x)^n dx \qquad (b) \int_0^1 (1-x^2)^n dx$$

4. Find the sum of the series $\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$. (10%)

5. Evaluate the integral $\int_0^1 \int_0^1 e^{\max\{x^2, y^2\}} dy dx$,

Where $\max\{x^2, y^2\}$ means the larger of the numbers x^2 and y^2 . (20%)

6. Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 6 & 10 & 15 \\ 1 & 4 & 10 & 20 & 35 \\ 1 & 5 & 15 & 35 & 70 \end{bmatrix}$$

Use Gaussian elimination to find the determinant of all upper left submatrices A_k , $k = 1, 2, 3, 4, 5$.

Note that $A_5 = A$. (10%)

(背面仍有題目，請繼續作答)

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7. Find the 3 by 3 symmetric matrix A and the sum and the product of all its eigenvalues. (10%)

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1^2 + 2x_2^2 + 7x_3^2 + 2x_1x_2 + 2x_1x_3 + 4x_2x_3.$$

8. (i) If A is a positive definite $n \times n$ matrix, and R is any real $n \times m$ matrix, what can you say about the definiteness of the matrix $R^T A R$? For which matrices R is $R^T A R$ positive definite?

(ii) If A is an indefinite $n \times n$ matrix, and R is any real $n \times m$ matrix of rank n , what can you say about the definiteness of the matrix $R^T A R$?

(iii) If A is an indefinite $n \times n$ matrix, and R is any real $n \times m$ matrix, what can you say about the definiteness of the matrix $R^T A R$? (10%)