系所組別：電信管理研究所乙組

考試科目：線性代數　　考試日期：0223，節次：2

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (1) Project $b = \begin{bmatrix}1\\1\\1\end{bmatrix}$ onto $a = \begin{bmatrix}1\\2\\2\end{bmatrix}$ to form the new vector p. (10%) (2) Find the projection matrix P onto a line through $a = \begin{bmatrix}1\\2\\2\end{bmatrix}$. (10%)

2. Find the eigenvalues and eigenvectors for matrices A and A^{∞}. Explain why A^{100} is close to A^{∞}. (20%)

   $A = \begin{bmatrix}0.6 & 0.2\\0.4 & 0.8\end{bmatrix}$

3. Every year, 2% of young people become old and 3% of old people become dead in one country. Suppose that the birth rate is zero (no births), what is the steady state of young, old and dead people for that country? Numerical proof is required to receive full credit. (20%)

4. What conditions on b_1, b_2, b_3, b_4 make each system solvable? (5%) What are the solutions of $\boldsymbol{x}$ in each system? (5%)

   (1) $\begin{bmatrix}1 & 2\\2 & 4\\2 & 5\\3 & 9\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix}$

   (2) $\begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\\2 & 5 & 7\\3 & 9 & 12\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix}$

5. Let x_a be the number of vehicles traveling through arc a ($\forall a \in A$) in an urban transportation network and $t_a(x_a)$ represents the relationship between x_a and the travel time for link a. Suppose that we are considering a network with huge number of vehicles, therefore x_a can be considered as continuous. (1) What is the implication of assuming $\frac{\partial t_a(x_a)}{\partial x_b} = 0 \ \forall a \neq b$ and $\frac{\partial t_a(x_a)}{\partial x_a} > 0 \ \forall a$? (10%) (2) Show that the function $\mathrm{z}(\mathbf{x}) = \sum_a \int_0^{x_a} t_a(\omega)\, d\omega$ is strictly convex. (20%)