1. The ODE of a damped mass-spring system is
$$my'' + cy' + ky = 0,$$
where m is the mass, c is the damping constant, and k is the spring constant. Find the general solutions if
(a) the damping constant c is so large that $c^2 > 4mk$, (5%)
(b) $c^2 = 4mk$, (5%)
(c) the damping constant c is so small that $c^2 < 4mk$. (5%)

2. Find the motion of the mass-spring system modeled by the ODE and the initial conditions:
$$y'' + 16y = 4\sin t, \quad y(0) = 1, \quad y'(0) = 1. \text{ (10\%)}$$

3. Find the motion of the damped mass-spring system under a unit impulse at time $t = 1$, modeled by
$$y'' + 4y' + 5y = \delta(t-1), \quad y(0) = 0, \quad y'(0) = 3. \text{ (10\%)}$$

4. $$A = \begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$
(a) Find the eigenvalues and the corresponding eigenvectors of A. (10%)
(b) Find a matrix X and its inverse X^{-1} such that $D = X^{-1}AX$ is diagonal. (10%)

5. Find a unit normal vector of the surface $6x^2 + 2y^2 + z^2 = 225$ at the point $P:(5,5,5)$. (5%)

6. Evaluate the integral $\iint_S \vec{F} \bullet \hat{n}\, dA$ over the surface S of $x^2 + y^2 \le 9$, $0 \le z \le 2$,

where $\vec{F} = [\sin y, \cos x, \cos z]$ and $\hat{n}$ is the outer unit normal vector of S. (10%)

7. A function is defined in a finite interval, $0 < x < 4$, such that $f(x) = 1$. Find
(a) the Fourier cosine series representation, (5%)
(b) the Fourier sine series representation. (5%)

8. Find the solution $u(x,t)$ of
$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad (0 < x < 1,\ t > 0)$$
with
$$u(0,t) = 0 \quad \text{and} \quad u(1,t) = 0 \quad (t > 0),$$
$$u(x,0) = 0.01\sin 3\pi x \quad \text{and} \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0 \quad (0 < x < 1). \text{ (10\%)}$$

9. Evaluate the integral
$$\oint_C \operatorname{Re} z\, dz,$$
where C consists of $|z| = 1$ counterclockwise in the complex z-plane. (10%)