國立中正大學101學年度碩士班招生考試試題

系所別：數學系統計科學　　　　科目：基礎數學

第1節　　　　

(20%) 1. Let $f(x) = 2x^{\frac{5}{3}} - 5x^{\frac{4}{3}}$, find the relative extremes of the function, the points of inflection, the intervals where the graph is concave upward and those where it is concave downward, and sketch the graph.

(10%) 2. (a) Use Lagrange multipliers to prove that the product of three positive number x, y, and z, whose sum has the constant value S, is a maximum when the three numbers are equal.

(b) Use (a) to prove that

$$\sqrt[3]{xyz} \leq \frac{x+y+z}{3}.$$

(10%) 3. Evaluate $\int_0^{\sqrt{\frac{\pi}{2}}} \int_x^{\sqrt{\frac{\pi}{2}}} \int_1^3 \sin(y^2) dz dy dx.$

(24%) 4. Let A be the 3×3 matrix given by

$$A = \begin{bmatrix} 5 & -1 & -1 \\ -1 & 5 & -1 \\ -1 & -1 & 5 \end{bmatrix}$$

(a) Find the eigenvalues and associated eigenvectors of A.

(b) Find an orthogonal matrix P, and a diagonal matrix Λ, such that

$$A = P\Lambda P^T.$$

(c) Find the inverse of A.

(d) Find the eigenspaces of A.

(e) Show that for the symmetric matrix the eigenspaces associated with different eigenvalues are orthogonal.

(18%) 5. Let V be the real vector space spanned by the rows of the matrix

$$A = \begin{bmatrix} 3 & 21 & 0 & 9 & 0 \\ 1 & 7 & -1 & -2 & -1 \\ 2 & 14 & 0 & 6 & 1 \\ 6 & 42 & -1 & 13 & 0 \end{bmatrix}$$

(a) Find a basis for V.

(b) Tell which vectors $(x_1, x_2, x_3, x_4, x_5)$ are elements of V.

(c) If $(x_1, x_2, x_3, x_4, x_5)$ is in V what are its coordinates in the basis chosen in part (a) ?

(18%) 6. Let T be the linear operator on R^3 defined by

$$T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3).$$

(a) What is the matrix of T in the standard ordered basis for R^3 ?

(b) What is matrix of T in the ordered basis

$$\{\alpha_1, \alpha_2, \alpha_3\}$$

where $\alpha_1 = (1, 0, 1)$, $\alpha_2 = (-1, 2, 1)$, and $\alpha_3 = (2, 1, 1)$?

(c) Prove that T is invertible and give a rule for T^{-1} like the one which defines T.