國立交通大學 103 學年度碩士班考試入學試題

科目：機率論(4082)　　考試日期:103年2月14日 第 2 節

系所班別：統計學研究所　　組別：統計所　　第 1 頁,共 1 頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. (20 points) How can 20 balls, 10 white and 10 black, be put into two urns so as to maximize the probability of drawing a white ball if an urn is selected at random and a ball is drawn at random from it?
2. (20 points) A coin that when flipped comes up head with probability p is flipped until either heads or tails has occurred twice. Find the expected number of flips.
3. (20 points) For some constants b and c, the random variable X with $EX = VarX$ has probability density function

$$f(x) = \begin{cases} cx^4 & 0 < x < b \\ 0 & otherwise \end{cases}$$

Find b and c.

4. The joint density function of X and Y is given by

$$f(x,y) = C(y-x)e^{-y} \qquad -y < x < y, \quad 0 < y < \infty$$

(a) (4 points) Find C.

(b) (4 points) Find the density function of X.

(c) (4 points) Find the density function of Y.

(d) (4 points) Find EX.

(e) (4 points) Find EY.

5. Suppose that X_i are independent Poisson random variables with respective means λ_i, $i = 1,2,3$. Let $X = X_1 + X_2$ and $Y = X_2 + X_3$. The random vector (X, Y) is said to have a bivariate Poisson distribution.

(a) (10 points) Find $Cov(X, Y)$.

(b) (10 points) Find the joint probability mass function $P(X = i, Y = j)$.