1. From two known stations with coordinates (E_1, N_1) and (E_2, N_2), and measuring the two horizontal angles to an unknown station (E_p, N_p), this scheme is known as space intersection. Please derive the error propagated from the observations to the station coordinates, assuming the measuring accuracy of these two angles is the same, σ_α, and the known coordinates are error free.

(a) List the observation equations, and the error propagation equations. (15%)

(b) Provide two numerical examples for illustrating the influence of intersection geometry. (15%)

2. Line l_1 passes through point (x_{11}, y_{11}, z_{11}) and point (x_{12}, y_{12}, z_{12}).

Line l_2 passes through point (x_{21}, y_{21}, z_{21}) and point (x_{22}, y_{22}, z_{22}).

Line l_3 passes through point (x_{31}, y_{31}, z_{31}) and point (x_{32}, y_{32}, z_{32}).

If these lines are not parallel, please provide a procedure to find the intersection of multi-rays. (20%)

3. Explain the concept of least-squares adjustment with conditional equations. Give a practical example (adjustment with conditional equations) in surveying. (10%)

4. The error covariance matrix of the estimated planar coordinates (X,Y) of a control point in a network is $\begin{bmatrix} 4 & 2 \\ 2 & 9 \end{bmatrix} \times 10^{-4} m^2$.

(a) What are the standard errors of X and Y? (5%)

(b) What is the correlation between X and Y? (5%)

(c) Compute and plot the semi-major and semi-minor axis of the error ellipse at the control point. (10%)

5. The elevations of a benchmark as determined by precision leveling near the coast of Yunlin are found to change over time as follows.

Time (years elapsed since a reference time)	Elevation (m)
0	4.7144
1	4.6750
2	4.6341
3	4.5972
4	4.5555

a. The elevation (y) is fitted by the simple linear model $y = c + dt$, where t is time. Find the coefficients c and d using least-squares method. (10%)

b. Predict the elevation at time = 5 years and its standard error using the result in (a). (10%)