

國立交通大學 103 學年度碩士班考試入學試題

科目：工程數學(3091)

考試日期：103 年 2 月 15 日 第 1 節

系所班別：土木工程學系

組別：土木系丙組一般生

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【可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考料是否相符!!

1. (20%) Find a complete solution of the linear differential system

$$\frac{dx}{dt} + 2x + \frac{dy}{dt} + 6y = 2e^t$$

$$2\frac{dx}{dt} + 3x + 3\frac{dy}{dt} + 8y = -1$$

2. (15%) Find the value of the surface integration

$$I = \iint_S (x^3 dydz + x^2 y dzdx + x^2 z dx dy)$$

where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ ($0 \leq z \leq b$) and the circular disks $z = 0$ and $z = b$ ($x^2 + y^2 = a^2$).

3. (15%) What is the directional derivative of the function $\varphi(x, y, z) = xy^2 + yz^3$ at point $(2, -1, 1)$ in the direction of the vector $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$?
4. (15%) Given a dynamic system equation $mu''(t) + cu'(t) + ku(t) = p(t)$, $t \geq 0$ with initial conditions $u(0) = u'(0) = 0$, where m , c and k represent respectively the mass, damping and stiffness of the system. Convert the system equation into frequency domain by Fourier transform and find the transfer function $H(\omega) = \frac{\hat{u}(\omega)}{\hat{p}(\omega)}$ where $\hat{u}(\omega)$ is the Fourier transform of $u(t)$ and $\hat{p}(\omega)$ is the Fourier transform of $p(t)$.
5. (20%) A one-dimensional heat equation is given by $u_t = c^2 u_{xx}$, $0 \leq x \leq 10$, $0 \leq t < \infty$. Please find the solution with non-homogeneous boundary conditions $u(0, t) = 0$, $u(10, t) = 20$ and an initial condition $u(x, 0) = \sin 3\pi x / 10$

$$(\text{NOTE: } b_n = \frac{2}{10} \int_0^{10} x \sin \frac{n\pi x}{10} dx = \frac{-20 \cos n\pi}{n\pi})$$

6. (15%) Find the Maclaurin series of $f(z) = \tan^{-1}(z)$. (NOTE: if $f(z) = \tan^{-1}(z)$, then $f'(z) = 1/(1+z^2)$)