國立交通大學 103 學年度碩士班考試入學試題

科目：工程數學(3051) 考試日期：103 年 2 月 15 日 第 1 節

系所班別：土木工程學系 組別：土木系甲組一般生 第 1 頁,共 2 頁

【可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. The earth surface is approximated as a sphere $x^2+y^2+z^2=R^2$ in Figure 1.
   (a) Write the parametric representation $r(u,v)$ of the sphere in terms of spherical coordinates u (longitude 經度) and v (latitude 緯度) where $-\pi \le u \le \pi$ and $-\pi/2 \le v \le \pi/2$. (5%)
   (b) China airlines flight CI032 from Taipei (121.23° E, 25.08° N) to Vancouver (123.13° W, 49.25° N) is scheduled to leave at 23:30 on January 17 (local time). Note that 123.13° W means $u=-123.13°$ and the radius of the earth R is 6,400 km.
      (b.1) Express the position vectors of Taipei and Vancouver in terms of the Cartesian coordinates. (5%)
      (b.2) Estimate the flying distance (mileage) of flight CI032 in km. (10%)
      (b.3) If the average flying speed of the flight is 850 km/hr, estimate the arrival time of CI032 at Vancouver. Note that the time of Vancouver behinds Taipei by 16 hours (5%)

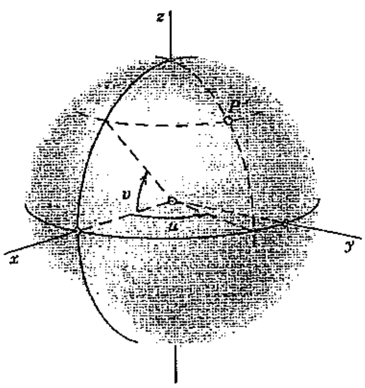

Figure 1

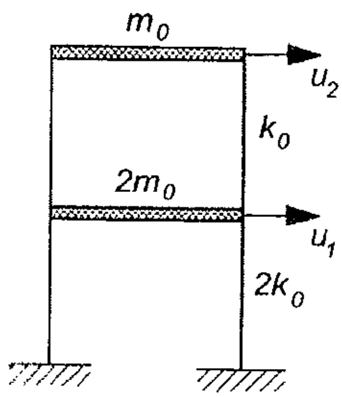

Figure 2

2. Consider a two-storey frame in Figure 2. The equation of motion of the frame in free vibration can be expressed as a 2nd order ordinary differential equation as

$$\begin{bmatrix} 2m_0 & 0 \\ 0 & m_0 \end{bmatrix}\begin{pmatrix} \ddot{u}_1(t) \\ \ddot{u}_2(t) \end{pmatrix}+\begin{bmatrix} 3k_0 & -k_0 \\ -k_0 & k_0 \end{bmatrix}\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

   (a) Find the natural frequencies (eigenvalues) and the corresponding mode shapes (eigenvectors) of the system if $m_0=k_0=1$ for simplicity. (10%)
   (b) Find the general solution of the dynamic system. (10%)
   (c) If the initial displacement of the system $\begin{pmatrix} u_1(0) \\ u_2(0) \end{pmatrix}=\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and initial velocity $\begin{pmatrix} \dot{u}_1(0) \\ \dot{u}_2(0) \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, find the dynamic response. (5%)

3. (15%) Solve the equation

$$[-x^2 \sin(xy) + 2y]dy + [\cos(xy) - xy\sin(xy)]dx = 0.$$

4. (15%) Let Σ be the piecewise smooth closed surface consisting of the surface Σ_1 of the cone $z = \sqrt{x^2 + y^2}$ for $x^2 + y^2 \leq 1$, together with the flat cap Σ_2 consisting of the disk $x^2 + y^2 \leq 1$ in the plane $z = 1$. Let $\vec{F}(x, y, z) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. Calculate the surface integral $\iint_\Sigma \vec{F} \cdot \vec{n} dS$ and the volume integral $\iiint_V \nabla \cdot \vec{F} dV$. Here $\vec{n}$ is the unit outer normal vector of Σ, and V is the set of points on and enclosed by Σ.

5. (20%) Solve the following partial differential equation by the method of separation of variables.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \text{ for } 0 < x < a, 0 < y < b,$$
$$u(x, 0) = 0, \text{ for } 0 \leq x \leq a,$$
$$u(0, y) = u(a, y) = 0, \text{ for } 0 \leq y \leq b,$$
$$u(x, b) = (a - x)\sin(x), \text{ for } 0 \leq x \leq a.$$