考試科目：工程數學

系所名稱：機械與機電工程學系碩士班不分組

1.答案以橫式由左至右書寫。2.請依題號順序作答。

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1. Solve the initial value problem: **(16%)**

$$3y^2 y' = 4x \; ; \quad y(0) = 1.$$

2. Solve the initial value problem: **(16%)**

$$x^2 y'' - 6y = 8x^2 \; ; \quad y(1) = 1, \quad y'(1) = 0.$$

3. Use the Laplace transform to solve the initial value problem: **(17%)**

$$y'' + y = 1 \; ; \quad y(0) = 6, \quad y'(0) = 0.$$

參考公式　　$\mathcal{L}[f''(t)](s) = s^2 F(s) - s f(0) - f'(0).$

4. Find the recurrence relation and use it to generate the first five terms of the Maclaurin series of the general solution. **(17%)**

$$y' - x^3 y = 1.$$

參考公式　Maclaurin series: $y(x) = \sum_{n=0}^{\infty} a_n x^n$

5. Find the eigenvalues and the corresponding eigenfunctions of the following problem on an interval $[0, \pi]$. **(17%)**

$$y'' + \lambda y = 0 \; ; \quad y'(0) = y(1) = 0.$$

6. Find the Fourier series of the function on the interval $[-\pi, \pi]$. **(17%)**

$$f(x) = -1 \quad \text{for} \quad -\pi \le x \le 0$$

$$f(x) = +1 \quad \text{for} \quad 0 \le x \le \pi.$$