東吳大學 103 學年度碩士班研究生招生考試試題

系級	數學系碩士班 A 組	考試時間	100 分鐘
科目	線性代數	本科總分	100 分

1. (30%) Let $A = \begin{bmatrix} 1 & -1 & 3 & 0 \\ -2 & 2 & 0 & 4 \\ -1 & 1 & 9 & 8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

   a. Reduce $[A|\mathbf{b}]$ to a row echelon form.

   b. Find the condition on $\mathbf{b}$ so that $A\mathbf{x} = \mathbf{b}$ is solvable.

   c. Find a basis for the column space of A.

   d. Find a basis for the left null space of A.

   e. Find the projection of $\begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix}$ onto the column space of A.

2. (20%) Diagonalize the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

3. (20%) Let the transformation T be a reflection across the line $y = x$ in the plane.

   a. Find the matrix M_{B_1} of T with respect to the basis $B_1 = \{(1,0), (0,1)\}$.

   b. Find the matrix M_{B_2} of T with respect to the basis $B_2 = \{(1,1), (1,-1)\}$.

   c. Show that M_{B_1} and M_{B_2} are similar.

4. (30%) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

   a. Show that A has linearly independent columns.

   b. Apply Gram-Schmidt process to find an orthogonal basis for the column space of A.

   c. Find the QR-factorization of A.

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