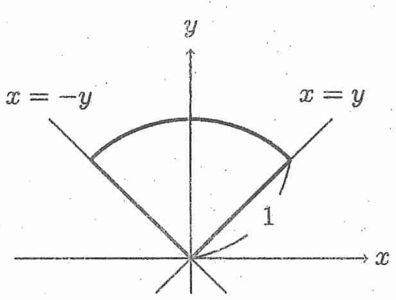
國立中央大學103學年度碩士班考試入學試題卷

所別：統計研究所碩士班 不分組(一般生) 科目：基礎數學 共 2 頁 第 1 頁
統計研究所碩士班 不分組(在職生)

本科考試可使用計算器，廠牌、功能不拘 ＊請在試卷答案卷（卡）內作答

1. (20%) Consider a double integral $\iint_R f(x,y)dxdy$ over a region R, where R is a sector of a circle with radius 1 as shown in the following figure.



   (a) (5%) Write down the integral explicitly in Cartesian coordinates (by finding the range of x and y).
   (b) (5%) Write down the integral explicitly in polar coordinates.
   (c) (10%) Consider the change of variables: $u = x - y$ and $v = x + y$. Find its Jacobian and rewrite the integral explicitly with respect to the new variables.

2. (25%) Let $f(x,y) = x^2 + y^3 - y$ be a smooth function on the region

$$R = \{x, y \in \Re^2 : -2 \le x \le y \le 2\}.$$

   (a) (3%) Describe the boundary of R.
   (b) (2%) Calculate the area of R.
   (c) (5%) Find the critical points of f over the interior of R.
   (d) (5%) Find the local minimum and local maximum of f over the interior of R.
   (e) (10%) Find the global minimum and global maximum of f over the interior and boundary of R.

3. (20%) Consider a sequence $a_0 = 1$, $a_1 = 1$ and $a_{n+2} = a_{n+1} + a_n$ for all $n > 1$. One can rewrite the recursive formula in terms of matrices as

$$\begin{pmatrix} a_{n+2} \\ a_{n+1} \end{pmatrix} = A \begin{pmatrix} a_{n+1} \\ a_n \end{pmatrix}$$

   where A is a 2×2 matrix.

   (a) (5%) Find A and its eigenvalues.
   (b) (5%) Find a general form of a_n in terms of eigenvalues and eigenvectors of A and n.
   (c) (10%) Find the minimal n so that $a_n \ge 10^{20}$.

參考用

注意：背面有試題

4. (35%) A linear transform P from a vector space V to itself is called a projection if $P^2 = P$. Suppose $V = \mathbb{R}^3$ and W is the subspace of V spanned by $(1, -1, 0)$ and $(1, 0, -1)$.

(a) (5%) Find all possible eigenvalues of P.

(b) (5%) Show that P is always diagonalizable.

(c) (5%) Find an orthogonal basis v_1, v_2, v_3 of V so that v_1 and v_2 is a basis of W.

(d) (10%) Find the matrix of the projection P on V given by $P(a_1v_1 + a_2v_2 + a_3v_3) = a_1v_1 + a_2v_2$.

(e) (10%) Consider a system of linear equations $X\beta = y$, where

$$X = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}, \quad \text{and } y = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

This system of linear equations indeed has no solutions. Find β that minimizes the sum of squared errors, $\mathrm{SSE}(\beta) = (X\beta - y)^t(X\beta - y)$. Here, the superscript t is matrix transpose. (Hint: Solve the equation $X\beta = Py$.)

參考用

注意：背面有試題