國立政治大學 103 學年度碩士班招生考試試題

考試科目	線性代數	所別	應數系 8111 8116	考試時間	2月23日(日)第二節

Please show all your work.

1. Prove that if W_1 and W_2 are finite-dimensional subspaces of a vector space V, then the subspace $W_1 + W_2$ is finite-dimensional, and $\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$.

2. Prove that the set of all $n \times n$ symmetric matrices forms a subspace of $M_n(\mathbb{R})$. Find the basis for this subspace.

3. Let T be a linear transformation from $M_n(\mathbb{R}) \to M_n(\mathbb{R})$ defined by

$$T(A) = \frac{A + A^T}{2}, A \in M_n(\mathbb{R}).$$

   Can T be diagonalized?

4. Find the eigenvalues, eigenvectors, the algebraic multiplicity and geometric multiplicity of each eigenvalue of the matrix

$$\begin{bmatrix} 3 & 0 & 4 & 4 \\ 0 & -1 & 0 & 0 \\ 0 & -4 & -1 & -4 \\ 0 & 4 & 0 & 3 \end{bmatrix}.$$

5. Let $V = P_4(\mathbb{R})$, and $W = span\{1, x\}$. Consider the inner product $< f, g >$ on V defined by

$$< f, g >= \int_0^1 f(x)g(x)dx, \ f, g \in V.$$

   What is the orthogonal projection of x^2 in W?

p.s. 20 points for each problem.

備註	試題隨卷繳交